Anomalous Coulomb correction to the scattering length in connection with the shift of the ground level of the K^-p atom

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It is shown for the case of an exactly solvable model that in an absorbing system the Coulomb correction to the scattering length is large where the scattering length itself is small. The anomalously large Coulomb correction to the $\bar{K}N$ scattering length is discussed as a possible explanation of the contradiction between the values of the KN scattering length obtained from low-energy $\bar{K}N$ scattering and those obtained from the shift of the ground level of the K^-p atom.

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Davies *et al.*[1] measured the shift of the ground level of the K^-p atom and found[1)]

$$\Delta E + \frac{i}{2}\Gamma = (40 \pm 60) + (0 + 115)i \text{ eV}.$$

This value of $[\Delta E + (i/2)\Gamma]$ corresponds to a small value of the $\bar{K}N$ nuclear-Coulomb scattering length, in contradiction of the $\bar{K}N$ scattering length obtained from low-energy $\bar{K}N$ scattering.[2,3] Deloff and Law[4] have reported detailed calculations of the Coulomb correction to the K^-p scattering length. They conclude that the Coulomb correction is too small to resolve this contradiction. Two hypotheses have been advanced to resolve the situation. According to one of them,[5,6] there is a near-threshold level in the KN system, and this level was not taken into account in the analysis of the scattering data. According to the other hypothesis,[4,7] the Coulomb correction may be amplified by nonlocal short-range effects. Neither of these possibilities has been pursued completely at this point, and we lack a satisfactory explanation for the data on the shift of the $1S$ level of the K^-p atom.

In this letter we wish to point out that inelastic effects substantially increase the Coulomb correction in a region where the real part of the nuclear-Coulomb scattering length is small. We know that in a system without absorption the opposite situation prevails: The Coulomb correction is large when the scattering length itself is large.[8] The simplest way to see the source of this important distinction is to examine the behavior of the scattering length as a function of a parameter, which is a measure of the depth of the nuclear potential. We write the short-range potential in the form $gV(r)$, where g is this parameter, which corresponds to the time at which the level appears. If there are no inelastic reactions, the scattering length becomes infinite at $g = g_0$. If we now introduce absorption, the behavior of the scattering length (more precisely, of its real part) at $g \sim g_0$ changes completely. Instead of becoming infinite, the real part of the scattering length crosses zero at $g \sim g_0$. This is the so-called Krell-Ericson phenomenon.[9,10] An important point here is that the zeros of Re A_s and Re

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A_{cs} (A_s and A_{cs} are the purely nuclear scattering length and the nuclear-Coulomb scattering lengths) are displaced from each other along the g axis, and the values themselves change rapidly near $g \sim g_0$. As a result, the Coulomb correction may prove very large where $\mathrm{Re}A_{cs}$ itself is small.

Let us test these arguments in the example of an exactly solvable model. Such a model was proposed in Ref. 11: There are two types of nonrelativistic spin-zero particles: "heavy" particles, which interact via the short-range and Coulomb potentials, and free "light" particles. A transition between the two types is described by a nondiagonal short-range potential. The short-range potential describing the interaction between the heavy particles and the potential describing the transition between channels are adopted in a simple separable form.

Denoting the heavy-particle channel as channel 1 and the light-particle channel as 2, we can thus write

$$V_{11} = V_s + V_c,$$
$$V_s = \eta\,|\xi\rangle\langle\xi|, \quad \langle r|\xi\rangle = \left(\frac{\xi}{2\pi}\right)^{1/2} \frac{\exp(-\xi r)}{r}, \tag{1}$$
$$V_c = \frac{2k\gamma}{r}, \quad \gamma = -(ka_B)^{-1}.$$

Here a_B is the Bohr radius of the system, given by $a_B = (\mu_1\alpha)^{-1}$, $\alpha = 1/137$, and μ_1 is the reduced mass of the heavy particles. We also have

$$V_{12} = \lambda\,|\beta\rangle\langle\beta|, \quad \langle r|\beta\rangle = \left(\frac{\beta}{2\pi}\right)^{1/2} \frac{\exp(-\beta r)}{r}. \tag{2}$$

It is convenient to introduce the dimensionless parameters

$$\eta_0 = -2\eta\mu_1\xi^{-2}, \quad \lambda_0^2 = 4\lambda^2\mu_1\mu_2\beta^{-4},$$
$$\kappa = (a_B\xi)^{-1}, \quad \nu = (a_B\beta)^{-1}. \tag{3}$$

In the single-channel problem with a separable potential V_s of the type in (1), a bound state appears at $\eta_0 = 1$. The scattering matrix for this model was constructed in Ref. 11:

$$A_s^{-1} = K_s - \tilde{\lambda}^2\left(\frac{2}{\beta}\right)\frac{\left(K_s + \dfrac{\xi\beta}{\xi+\beta}\right)^2}{1 + \tilde{\lambda}^2\dfrac{2}{\beta}\left(K_s + \dfrac{2\xi\beta}{\xi+\beta} - \dfrac{\beta}{2}\right)}, \tag{4}$$

$$A_{cs}^{-1} = K_{cs} - \tilde{\lambda}^2\left(\frac{a_B}{2\pi}\right)\frac{v_{\nu\nu}\left(K_{cs} + \dfrac{2\pi}{a_B}\dfrac{u_{\kappa\nu}}{v_{\kappa\nu}}\right)^2}{1 + \tilde{\lambda}^2\dfrac{a_B}{2\pi}v_{\nu\nu}\left(K_{cs} + \dfrac{4\pi}{a_B}\dfrac{u_{\kappa\nu}}{v_{\kappa\nu}} - \dfrac{2\pi}{a_B}\dfrac{u_{\nu\nu}}{v_{\nu\nu}}\right)}. \tag{5}$$

Here

$$K_s = -\frac{\xi}{2}\left(1-\frac{1}{\eta_0}\right),$$

$$K_{cs} = -\frac{\xi}{2}e^{4\kappa}\left(1-\frac{1}{\eta_0}\right)-\frac{2}{a_B}\,\mathrm{Re}\,\Gamma(0,-4\kappa),$$

$$\tilde{\lambda}^2 = \lambda_0^2(1-i\rho)^{-2}, \tag{6}$$

$$\rho = \frac{(2\mu_2 Q)^{1/2}}{\beta}, \qquad Q = m_1 + M_1 - m_2 - M_2,$$

$$v_{\kappa\nu} = 4\pi(\kappa\nu)^{1/2}\exp(-2\kappa-2\nu),$$

$$u_{\kappa\nu} = 2(\kappa\nu)^{1/2}\{(\kappa+\nu)^{-1} + 2\exp(-2\kappa-2\nu)\mathrm{Re}\,\Gamma(0,-2\kappa-2\nu)\}.$$

Expressions (4) and (5) can be used to test the hypothesis that the Coulomb correction $\mathrm{Re}(A_s - A_{cs})$ may be anomalously large in the critical region (i.e., at $\eta_0 \sim 1$). We choose the parameters in (4) and (5) to correspond to the problem of the K^-p atom. We set $a_B = 84$ F, and we set the masses of the heavy and light particles equal to the masses of the particles in the K^-p and $\Sigma\pi$ channels, respectively. The parameters ξ, β, and λ_0^2 were varied over the ranges

$200\ \mathrm{MeV} \leqslant \xi \leqslant 400\ \mathrm{MeV}$

$300\ \mathrm{MeV} \leqslant \beta \leqslant 1000\ \mathrm{MeV}$

$0.1 \leqslant \lambda_0^2 \leqslant 1.$

Figures 1–3 show some typical results on $\mathrm{Re}A_s$ and $\mathrm{Re}A_{cs}$ vs the parameter λ_0, which is a measure of the depth of the diagonal potential in the heavy-particle channel. It can be seen from these figures that near the zero of $\mathrm{Re}A_{cs}$ the Coulomb correction is very large: $\mathrm{Re}A_s$ and $\mathrm{Re}A_{cs}$ differ from each other either by an order of magnitude or in sign.

A general feature of the curves in Figs. 1–3 is that the scattering length $\mathrm{Re}A_s$ is positive in the region where $\mathrm{Re}A_{cs}$ is approximately zero. This situation is opposite that which holds in the K^-p atom.[1] To obtain the necessary relationship between $\mathrm{Re}A_s$ and $\mathrm{Re}A_{cs}$ we should add a slight complication to the model. In the separable potential V_s used above, of the type in (1), there is only a single bound state, so that $\mathrm{Re}A_s$ and $\mathrm{Re}A_{cs}$ cross zero once. If the potential V_s gave rise to a second level,[2)] then the scattering length $\mathrm{Re}A_s$ would be negative near the second zero of $\mathrm{Re}A_{cs}$, as can be seen clearly from Figs. 2 and 3.

In these calculations a typical value of $\mathrm{Im}A_{cs}$ in the region with $\mathrm{Re}A_{cs} \simeq 0.1$ F was $\mathrm{Im}A_{cs} \sim 1$–3 F, or much larger than follows from the experimental value which we

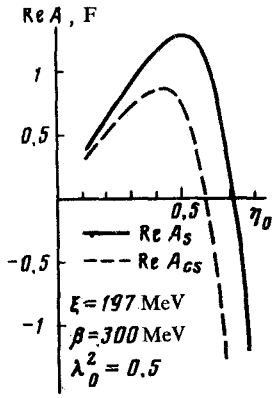

FIG. 1.

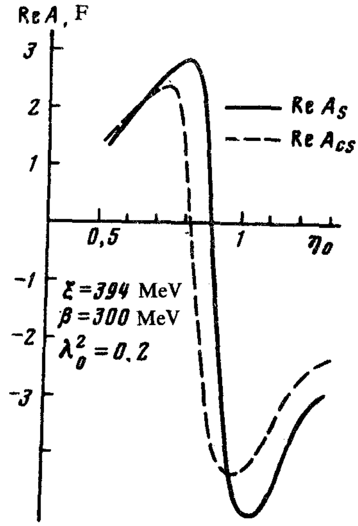

FIG. 2.

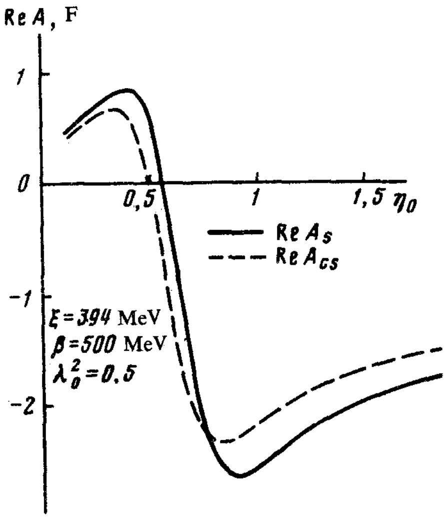

FIG. 3.

FIG. 1–3. Real part of the scattering length vs a parameter which is a measure of the depth of the short- range potential.

cited at the beginning of this letter. It is also difficult to explain the small value of $\mathrm{Im}A_{cs}$ in the models which assume the existence of a near-threshold resonance.[6] We might also note that $\mathrm{Im}A_{cs}$ is slightly higher according to other experimental results,[12]

In summary, the Coulomb correction to the scattering length can be anomalously large when absorption is taken into account.

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[1)]The sign of the real part of the shift is chosen in accordance with the definition $\Delta E = E_0 - \mathrm{Re}E_{\mathrm{exp}}$, so that a positive ΔE corresponds to an effective attraction.

[2)]The first, and deeper, level could be (for example) the well-known resonance Y_0^* (1405).

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