Inversion of hot carriers in Landau levels

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A method is proposed for creating an inverted population of hot carriers in Landau levels of the light subband in crossed **E** and **B** fields. Impurity scattering in this case facilitates inversion and for it "ultrapure" semiconductors are not required.

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Distributions of hot current carriers in semiconductors in the presence of inelastic scattering by optical phonons (OP) are distinguished by their unusual properties[1] and attract attention due to the possibility of obtaining inverted distributions,[2,3] which can be used to generate and amplify electromagnetic radiation in the far infrared and

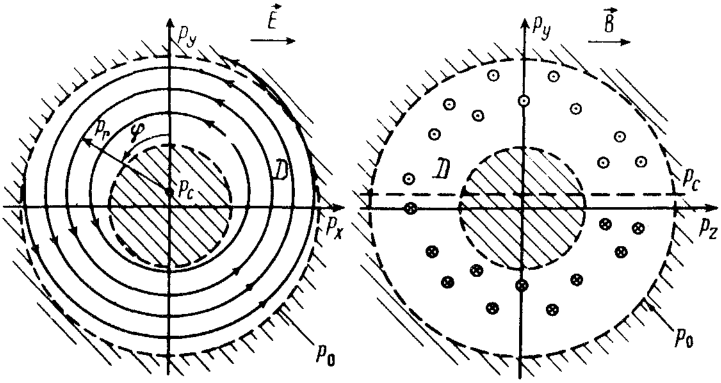

FIG. 1. Cyclotron rotation trajectories in momentum space and region of accumulation D with low collision frequency.

submillimeter regions.[4] At present, methods are known for obtaining inversion with respect to energies and crossed electric **E** and magnetic **B** fields,[3,5] leading to the experimental discovery of inverted distributions.[6,7] At the same time, the possibility of creating inversions in the Landau levels in real semiconductors is still doubted[2,8] due to the influence of impurity scattering. In this paper, we propose an efficient mechanism for creating inversion in the Landau levels of the light subband, which can lead to maser action (see also Ref. 9).

The mechanism is based on the large magnitude of both impurity scattering and scattering by OP in semiconductors with degenerate subbands of heavy and light holes. It consists of the following. The decrease in impurity scattering with increasing energy ϵ and subsequent increase in the collision frequency ν due to spontaneous emission of OP leads to the appearance of a region D in momentum space with a low collision frequency. This region is situated in the vicinity of a sphere $p = p_{0i}$ beneath OP (in Fig. 1, the region D is shaded), where $p_{0i} = (2m_i\hbar\omega_0)^{1/2}$, m_i is the effective mass, $\hbar\omega_0$ is the energy of the OP, and the index $i = \hbar$ for heavy holes and $i = l$ for light holes. The lifetime (time between collisions with exit from D) of current carriers (CC) in the D region is large compared to the lifetime of CC outside this region and, for this reason, if the source of CC, which appears after scattering, overlaps the D region and the carriers do not leave it during cyclotron rotation, then CC will accumulate in the D region, leading to inversion in the Landau levels.

In momentum space the trajectories of free CC motion in **E**⊥**B** fields are circles centered at $p_{ci} = m_i cE/B$ (see Fig. 1), while the quasiclassical analog of the Landau-level number is the energy of cyclotron rotation $\epsilon_{ri} = p_{ri}^2/2m_i$, where $p_{ri}^2 = p_x^2 + (p_y - p_{ci})^2$, and c is the velocity of light. The kinetic equation for the distribution functions of CC $f_i(p_r,\phi,p_z)$ in both subbands has the form

$$\omega_{Bi}\,\partial f_i/\partial\phi = -\nu_i f_i + I_i \qquad (1)$$

where $\nu_i = \Sigma_j \nu_{ij}$; $I_i = \Sigma_j I_{ji}$; $\nu_{ij} = \int w_{ji}(\mathbf{p},\mathbf{p}')d^3\mathbf{p}'$ is the collision frequency with transition out to the ith into the jth subbands; $I_{ji} = \int w_{ij}(\mathbf{p}',\mathbf{p})f_j(p')d^3\mathbf{p}'$ is the source of CC in subband i, appearing from the jth subband as a result of scattering; $\omega_{Bi} = qB/m_i c$ are the cylcotron frequencies, and q is the CC charge. In the active energy range $(\epsilon > \hbar\omega_0)^1$ at lattice temperatures lower than the Debye temperature the main scattering mechanism for CC is spontaneous emission of OP and here the collision frequency ν is large; thus, for deformation scattering, $\nu_{ij} = 2\nu_{0j}(\epsilon/\hbar\omega_0 - 1)^{1/2}$, where the constants ν_{0j} are determined by the parameters of the deformation potential.[10] For impurity scattering (main scattering in the passive energy range $\epsilon < \hbar\omega_0$) a sharp drop in ν with increasing ϵ is characteristic, for example, $\nu_{Ih} = \nu_I(\epsilon_I/\epsilon)^{3/2}/(1 + \epsilon_I/\epsilon)^2$, where the constants ν_I and ϵ_I are determined by the concentration of ionized impurities.[10]

For free motion, CC remain in D if the fields $\mathbf{E}$ and $\mathbf{B}$ are such that $p_{ci} \ll p_{0i}$; in this case the inner trajectories (trajectories lying in the passive region) almost entirely fill D (see Fig. 1). Here almost the entire phase volume of the region D will be used to accumulate CC (in contrast to accumulation of CC in the spindle-shaped region around p_{ci} (compare Refs. 3 and 5). Scattering of CC in the light subband ($i = l$) occurs with predominant transition to the heavy ($i = h$) subband due to its large density of states and, for this reason, for the subband l there exists an efficient mechanism for eliminating CC (not located in D) due to scattering. If for subband l the condition $p_{cl}/p_{0l} \ll 1$ is satisfied, while in the heavy subband p_{ch}/p_{0h} is not too small (we assume that $m_l < m_h$), then for $\nu_{Eh} \sim \nu_{0h}$ a wide source I_{lh} is created in the subband l due to intraband scattering of heavy CC, penetrating deeply into the active region and emitting OP; here $\nu_{Eh} = qE/p_{0h}$ is the transit frequency. For $p_{cl} \ll p_{0l}$ and $\omega_{Bl} \gg \nu_l$ to the dependence of f_l and ϕ is quite smooth, so that after averaging (1) with respect to ϕ, we obtain

$$<f_l>_\phi = \int f_l \, d\phi / 2\pi = <I_l>_\phi / <\nu_l>_\phi . \tag{2}$$

It follows immediately from this expression that for sufficiently wide I_l, in order to have an inversion in the Landau levels, it is necessary that the dependence of $\langle \nu_l \rangle_\phi$ on p_r have a minimum for large p_r, i.e., the inversion arises due to accumulation of carriers in D with long lifetimes $\tau \approx \langle \nu_l \rangle_\phi^{-1}$. The intraband transitions due to impurity scattering in the subband l with $p_{cl} \ll p_{0l}$ do not change the structure of the distribution function f_l, in the form of a spherical layer schematically illustrated by the trajectories in Fig. 1, since impurity scattering conserves energy and does not remove CC from D. This means that here there is also an inversion in energy.[11] For $p_{cl} \ll p_{0l}$, interband scattering (not necessarily impurity scattering), due to the principle of detailed balance, transfers the energy inversion in the heavy subband[3] into inversion in the Landau levels of the light subband.

We shall demonstrate the appearance of an inverted population of Landau levels for the light subband of p-Ge. The dependences of $\langle \nu_l \rangle_\phi$ on p_r with $p_z = 0$ are shown in Fig. 2 for several scattering mechanisms: impurity (im) and optical (op) and acoustical (ap) phonons ($T = 20$ K). It is evident from this figure that the collision frequency has a minimum for $0.3 \leqslant p_r/p_{0l} \geqslant 0.8$. Figure 3 shows the distribution function $\langle f_l \rangle_\phi$, calculated by the Monte Carlo method. These calculations show that for $p_z = 0$ there is a distinct inversion in the Landau levels, which correlates with $\langle \nu_l \rangle_\phi$; the most populated trajectories are those with p_r, where $\langle \nu_l \rangle_\phi$ is minimum. For the conditions

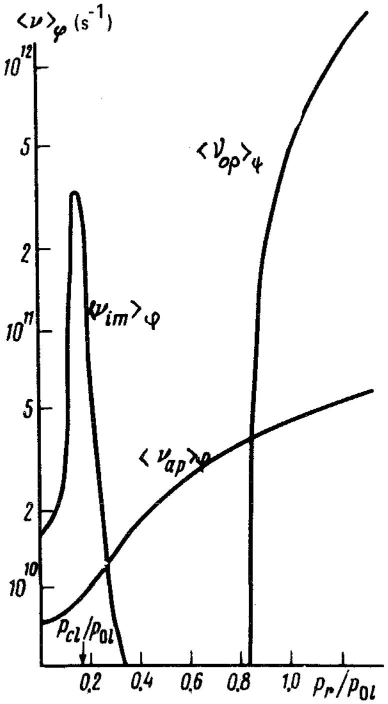

FIG. 2. Collision frequencies of light holes in p-Ge, averaged over the phase of cyclotron rotation ϕ: $N_I = 10^{14}$ cm^{-3}; $T = 20$ K; $p_{cl}/p_{0l} = 0.17$; $p_z = 0$.

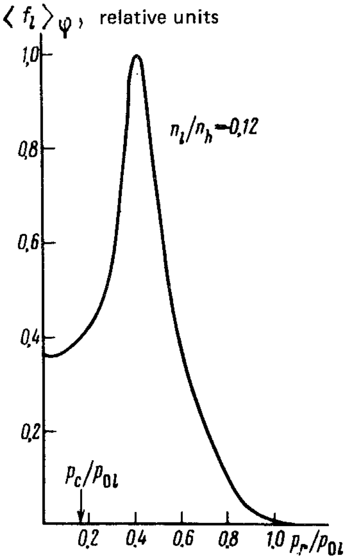

FIG. 3. Section of the distribution function $\langle f_l\rangle_\phi$ for $p_z = 0$; $N_I = 10^{14}$cm^{-3}; $T = 20$ K; $E = 1750$ V/cm; $B = 18$ kG; $p_{cl}/p_{0l} = 0.17$.

examined, the ratio of concentrations of heavy and light holes $n_l/n_h \approx 0.12$, i.e., approximately three times greater than the equilibrium value is $(m_l/m_h)^{3/2}$.

The appearance of an inversion in the Landau levels, which must lead to a number of interesting physical effects,[2,11] can easily be observed experimentally from the stimulated or spontaneous radiation, as well as from measurements of absorption (amplification) in the cyclotron resonance of light holes. The population inversion in Landau levels can be separated and used due to the nonequidistant nature or nonuniformity of the broadening of the levels in the degenerate nonparabolic or nonspherical subbands.

As we found out, stimulated emission, involving inversion of the population of Landau levels of the light subband under conditions close to those discussed in this paper, was recently observed experimentally by Yu. L. Ivanov and Yu. B. Vasil'ev (a private communication).

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