Stability of filaments of discotic liquid crystals

V. G. Kamenskiĭ and E. I. Kats
L. D. Landau Institute of Theoretical Physics, Academy of Sciences of the USSR

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The stability of a cylindrical specimen of a discotic liquid crystal is investigated. The critical value of the radius, beginning at which the cylindrical shape is stable, is found. For smaller radii, the maximum permissible length of the cylinder is determined.

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In recent years, appreciable progress has been made in the study of the properties of films of smectic liquid crystals,[1] due primarily to the assimilation of the technology for fabricating thin (up to two molecular layers) and homogeneous free (without a substrate) films. An analogous study of discotic liquid crystals is retarded by difficulties of obtaining free thin filaments. Thus, in Ref. 2, it was not possible to obtain specimens containing less than 400 molecular filaments. Below, we shall demonstrate that these difficulties are of a fundamental nature and we shall also give an estimate of the possible dimensions of specimens with the characteristic values of parameters of liquid crystals.

The problem of the stability of a fluid cylinder was solved by Rayleigh[3] and in a more general formulation by Bohr.[4] It was shown that a cylindrical specimen of an isotropic fluid is unstable relative to separation into pieces with a length of the order of ten radii. This instability is related in an obvious manner to the surface tension by the circumstance that the cylindrical shape does not minimize the surface energy. It is important to note that the instability occurs for any radius of the fluid cylinder.

For a cylinder formed by a discotic liquid crystal, whose axis coincides with the orientation of the filaments, there are two factors that determine stability. First, as in the case of an isotropic fluid, there is the surface energy which is minimized by separation of the cylinder. Second, there is the energy of elastic deformation of the lattice of fluid columns, which resist the decrease in the radius of the cylinder.

To estimate the effect under examination, we shall use the approximation of an isotropic fluid and viscosity, and we shall ignore the thermal conductivity. The equations of hydrodynamics for an incompressible discotic liquid crystal in the form of a cylinder in this approximation have the following form[5,6]:

$$\frac{\partial v_z}{\partial z} + \frac{1}{r}\frac{\partial}{\partial r}(r v_r) = 0,$$

$$\frac{\partial u_r}{\partial t} - v_r = \gamma B \frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r u_r)\right],$$

$$\rho\frac{\partial v_r}{\partial t} = -\frac{\partial P}{\partial r} + B\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r u_r)\right] + \eta\left[\frac{\partial^2 v_r}{\partial z^2} + \frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial}{\partial r}(r v_r)\right)\right], \qquad (1)$$

$$\rho\frac{\partial v_z}{\partial t} = -\frac{\partial P}{\partial z} + \eta\left[\frac{\partial^2 v_z}{\partial z^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right)\right].$$

Here the z axis of the cylindrical coordinate system is chosen along the axis of the cylinder, v is the velocity, u is the displacement vector of the lattice of fluid columns, P is the pressure, ρ is the density, η is the effective viscosity (some combination of Leslie coefficients), B is the isotropic elastic modulus of the lattice, and γ is the permeation factor, which describes the motion with a fixed lattice.

The complete solution of the system (1) can be found by expanding the radial dependences with respect to Bessel functions. We shall, however, take advantage of the fact that the cylinder is quite thin, i.e., its length L is much greater than its radius R. It follows from the first equation of the system (the continuity equation) that

$$v_z \sim v_r\frac{L}{R} \gg v_r$$

Ignoring, where possible, the radial-velocity component, as well as the change in the axial component of the velocity in the radial direction, we seek the solution in the following form:

$$\left.\begin{array}{l} v_z = \varphi_1(r) \\ v_r = \varphi_2(r) \\ u = \varphi_3(r) \\ P = \psi(r) \end{array}\right\} \times e^{iqz+\alpha t} \tag{2}$$

Substituting (2) into (1) and solving the system, we obtain

$$\varphi_1 = \frac{2iC}{q}, \quad \varphi_2 = Cr, \quad \varphi_3 = \frac{Cr}{\alpha}, \quad \psi = -\frac{2(\alpha\rho + \eta q^2)C}{q^2}, \tag{3}$$

where the constant C is determined from the boundary conditions.

The boundary conditions in this case are the conditions for finiteness and continuity along the axis of the cylinder, already included in (3), as well as the conditions for the stress tensor at $r = R$:

$$\sigma_{rz} = 0, \quad \sigma_{rr} = P - B\frac{1}{r}\frac{\partial}{\partial r}(ru_r) = P_\sigma, \tag{4}$$

where

$$P_\sigma = -\frac{\sigma}{R^2}\left(\xi + R^2\frac{\partial^2 \xi}{\partial z^2}\right),$$

σ is the surface tension coefficient, and ξ is the displacement of the lateral surface of the cylinder ($\partial\xi/\partial t = v_r$). Substitution of the solution (3) into the boundary conditions (4) yields the dispersion equation for α,

$$\alpha^2 + \frac{\eta q^2 \alpha}{\rho} - \left[\frac{\sigma q^2}{2\rho R}(1 - q^2R^2) - \frac{Bq^2}{\rho}\right] = 0.$$

The instability (i.e., the growing perturbation) occurs for $\alpha > 0$. The size of the regions of instability for fixed-cylinder radius is determined by the wave vectors $q_{\max}$, at which α is maximum. For low viscosities,

$$L \sim \frac{2\pi}{q_{max}} = \frac{9.02R}{\sqrt{1 - 2BR/\sigma}} \tag{5}$$

For high viscosities $\eta \gg \sqrt{\sigma R \rho/2}$, characteristic for discotic crystals,

$$L \sim 13\sqrt[4]{\frac{\eta^2 R^3}{4\rho\sigma}} \bigg/ \sqrt{1 - \frac{2BR}{\sigma}}. \tag{5'}$$

We note that for $2BR/\sigma > 1$ the cylinder of a discotic liquid crystal is generally stable.

We shall estimate the maximum possible filament length for specimens used in Ref. 2. In this case,

$R \simeq 750$ Å, $B \sim 10^6$ erg/cm^3, $\sigma \sim 10^2$ erg/cm^2, $\rho \sim 1$ g/cm^3, $\eta \sim 1$ P.

Substituting these values into (5′), we obtain $L \sim 2 \times 10^{-3}$ cm, which agrees with the size 20 μm found in Ref. 2.

In the above calculations we ignored gravity, which imposes certain restrictions on the characteristic length $L < \max\{\sigma/g, B/\rho g\} \sim (1-10^3)$ cm. The estimates made above show that the maximum attainable filament lengths more than satisfy this inequality.

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