Possible bound state of a K^- meson with a ^{4}He nucleus

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The analytic theory of nuclear level shifts developed in some earlier papers is applied to the ^{4}He K^- atom. The experimental data on the shift of the $2p$ level indicate that there may be a weakly bound state in the ^{4}He K^- system. The binding is "weak" on the nuclear scale; the binding energy is $\epsilon \sim 100$ keV. The probabilities for radiative transitions to this level are calculated.

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Three experimental teams, working independently, have recently reported clear indications of an anomalously large shift of the $2p$ level in the ^{4}He K^- atom (see the review by Batty[3]). The shift is $\Delta E_{2p} = E_{2p} - E_{2p}^{(0)} > 0$; i.e., the $2p$ level is pushed upward. Table I shows the experimental shifts and the widths of the $2p$ level, taken from Ref. 3. Calculation of these parameters from the optical potential[2)]

$$V_{opt}(r) = \frac{2\pi}{m}\bar{a}\rho(r) \tag{1}$$

yields[3] $\Delta E_{2p} = 0.2$ eV and $\Gamma_{2p} = 2$ eV, more than an order of magnitude away from the experimental values of ΔE_{2p} and Γ_{2p}. Anomalously large shifts of atomic levels

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TABLE I.

Nº	ΔE_{2p}, eV	Γ_{2p}, eV	ϵ, keV	$\gamma/2$, keV	$a_1^{(cs)}$, F^3
1	35 ± 12	30 ± 30	94	44	380 − 160 *i*
2	50 ± 12	100 ± 40	35	38	540 − 530 *i*
3	43 ± 8	55 ± 34	62	43	460 − 290 *i*

Note: The quantities ϵ, $\gamma/2$, and $a_1^{(cs)}$ are calculated for the average shift and width of the $2p$ level, without consideration of the experimental errors.

usually occur because there is a (real or virtual) level near zero in a strong potential $V_s(r)$ (see Ref. 1, for example). Let us examine the situation in the ^{4}HeK^- atom from this standpoint.

Experiments on hadronic atoms are conveniently analyzed by working from the equation[2]

$$\prod_{j=1}^{l}\left(\frac{\zeta^2}{j^2}-\lambda^2\right)\{\lambda+2\zeta[\psi(1-\zeta/\lambda)+\ln\lambda/|\zeta|]\}=\frac{1}{a_l^{(cs)}}+\frac{1}{2}r_l^{(cs)}\lambda^2, \tag{2}$$

which relates the shifts and widths of the atomic levels to the low-energy scattering characteristics. Here $\lambda=(-2E/E_C)^{1/2}$, E is the level energy, l is the angular momentum, $\zeta=-Z_1Z_2$, $a_l^{(cs)}$ is the nuclear-Coulomb scattering length, and $r_l^{(cs)}$ is the effective radius. For the ^{4}HeK^- system we have $\zeta=2$, a reduced mass $m=436.0$ MeV, a length unit $L=2a_B=62.0$ F, and an energy unit $E_C=23.2$ keV.

The effective radius $r_1^{(cs)}(l=1)$ in (2) is calculated in the following manner. The density of nucleons in an α particle can be described well by the Gaussian distribution $\rho(r)=C\exp(-r^2/r_0^2)$; the rms charge radius is $\langle r_{ch}^2\rangle^{1/2}=r_0(3/2)^{1/2}=1.67$ F (Ref. 4). According to (1), the interaction potential V_s acting between the K^- meson and the ^{4}He nucleus is of the same form. It can be shown that we have $r_1^{(s)}=-2.06/r_0$ for a Gaussian potential V_s (with the Coulomb force turned off) or

$$r_1^{(s)}=-c_1/<r_{ch}^2>^{1/2} \tag{3}$$

($c_1=2.52$). Introducing a Coulomb correction to $r_1^{(s)}$ in accordance with Eq. (4) of Ref. 5, we find $r_1^{(cs)}$. Using the value of $\langle r_{ch}^2\rangle^{1/2}$, we find[3] $r_1^{(s)}=-94.8L^{-1}=-1.53\ F^{-1}$, $r_1^{(cs)}=-120L^{-1}$. The Coulomb renormalization of the effective radius is quite significant here; this is a distinctive feature of the p wave.[5]

When we adopt a different model for $V_s(r)$, the only change in (3) is in the coefficient c_1. For a square well, for example, we would have $c_1=2.32$ and $r_1^{(s)}=-87.3$, while for the experimental potential we find $c_1=2.89$ and $r_1^{(s)}=-108.6$. We accordingly varied the parameter $r_1^{(s)}$ from -85 to -100 in calculating the spectrum of the ^{4}HeK^- atom. For a given value of $r_1^{(cs)}$, Eq. (2) determines the p-wave scattering length $a_1^{(cs)}$ and also the positions of the other p levels.

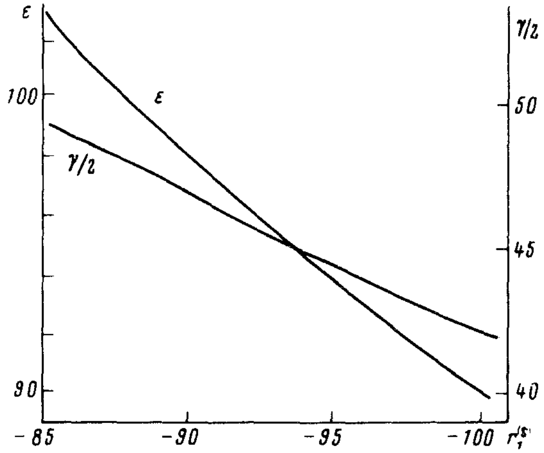

FIG. 1. Binding energy and half-width of the nuclear level vs the effective radius $r_1^{(s)}$. These calculations were carried out for version No. 1 in Table I. The values of ϵ and $\gamma/2$ are in keV, and $r_1^{(s)}$ is expressed in units of L^{-1}.

It turns out that in addition to the upward-shifted atomic and p levels ($n = 2, 3, \ldots$) the system contains a deeper nuclear level, which distorts the Coulomb spectrum. Its position depends strongly on the shift ΔE_{2p} and, less strongly, on the radius $r_1^{(s)}$. Table I shows the binding energy ϵ and the width γ of this state, along with the scattering length $a_1^{(cs)} \approx a_1^{(s)}$. Figure 1 shows how the calculated results depend on the parameter $r_1^{(s)}$.

It can be seen from Table I that the experimental position of the $2p$ level in the ^{4}He K^- atom cannot yet be regarded as established. As a result, there are significant differences between the values of ϵ and γ calculated[4)] from Eq. (2) for several versions of ΔE_{2p} and Γ_{2p}. Nevertheless, our calculations show that with $l = 1$ the ^{4}He K^- system should have a bound state with a binding energy on the order of a few tens of keV. The average radius of this state at $\epsilon \sim 100$ keV is 2–2.5 times $\langle r_{ch}^2 \rangle^{1/2}$.

A direct test of the existence of this nuclear level would be to look for radiative transitions to it from atomic levels. In the dipole approximation, transitions from s and d levels are possible, but in transitions of the K^- meson from high orbitals the d levels, rather than s levels, are populated. Ignoring the effect of the Coulomb force on the wave function of the nuclear level, we find the transition probability to be

$$w(nd \to \nu p) \approx \omega_0 \frac{8\nu^2}{15n^3}\left(1 - \frac{1}{n^2}\right)\left(1 - \frac{4}{n^2}\right)|(r_1^{(s)} a_B)|^{-1} , \tag{4}$$

where $\omega_0 = (\alpha^3 E_C/\hbar)\zeta^4[(Z_1 m_2 - Z_2 m_1)/(m_1 + m_2)]^2 = 0.179$ eV, and $\nu = \zeta/\lambda$ corresponds to the energy of the nuclear level (in this case, $\nu \sim 0.7$–1). Estimates from this expression yield $w(3d \to \nu p) \sim 3 \times 10^{-5}$ eV and a ratio $w(3d \to \nu p)/w(3d \to 2p) \sim (2$–$5) \times 10^{-2}$. We might note that the values given for ΔE_{2p} and Γ_{2p} in Table I were in fact found from measurements of $nd \to 2p$ radiative transitions.

Another way to observe the $^4He\,K^-$ nucleus would be to study nuclear reactions involving K^- mesons and light nuclei. In the reaction $K^- + {}^6Li \to d + X$, for example, this $^4He\,K^-$ level might be seen as a peak in the missing-mass spectrum for forward-emitted deuterons.

At first glance, the existence of a level with a binding energy $\epsilon \sim 100$ keV might seem surprising, since the shift of the $2p$ level in the $^4He\,K^-$ atom is relatively small: $\delta = \Delta E_{2p}/(E^{(0)}_{3p} - E^{(0)}_{2p}) \sim 7\times 10^{-3}$. We can state a general criterion for the possible existence of a weakly bound state. Working from the perturbation-theory expression[6] for the level shift, and assuming that $a_l^{(s)}$ is equal to the scattering length for a hard sphere of radius r_0, we find the "critical" value of the parameter $\delta = |E_{nl} - E^{(0)}_{nl}|/(E^{(0)}_{n+1,l} - E^{(0)}_{n,l})$ to be

$$\delta_{cr}^{(nl)} \approx \frac{(n+l)!}{(2l)!\,(2l+1)!\,(n-l-1)!}\left(\frac{2r_0}{na_B}\right)^{2l+1}. \tag{5}$$

With increasing l, the critical value δ_{cr} falls off rapidly. With $r_0/a_B = 1/20$ and $n = l+1$, for example, we find $\delta_{cr} \approx 0.1$, 10^{-4}, and 10^{-9} for $l = 0$, 1, and 2, respectively. If $\delta \gg \delta_{cr}$, the perturbation of the atomic spectrum should be regarded as strong, and the system should have a weakly bound nuclear state [whose position in the case $r_0 \ll a_B$ is given by Eq. (2)]. We see that the condition $\delta \gg \delta_{cr}$ holds in the $^4He\,K^-$ atom. The condition $\delta \gtrsim \delta_{cr}$ is a quick test of whether "nuclear" levels exist in various hadronic atoms.

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[2)]Here $\rho(r)$ is the density of the nuclear matter, and $\bar{a}$ is the effective KN scattering length, fitted on the basis of the shifts and widths of the levels of heavier kaonic atoms. We are using atomic units: $\hbar = m = e = 1$, where m is the reduced mass. The first Bohr radius of the system is $a_B = |\zeta|^{-1}$, and the energies of the unperturbed Coulomb levels are $E^{(0)}_{nl} = -\zeta^2/2n^2$ (in units of $E_C = me^4/h^2$).

[3)]We recall that in the case $l = 1$ the "radii" $r_1^{(s)}$ and $r_1^{(cs)}$ have the dimensions of a reciprocal length and are negative (if there is a level near zero in the potential V_s).

[4)]Equation (2) is itself very accurate, since the problem has a small parameter, $r_0/a_B \sim 1/20$. The uncertainty regarding $r_1^{(s)}$ generates only a 10% variation in ϵ and γ (Fig. 1).

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