Topological susceptibility in a SU(3) lattice gauge theory

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The topological susceptibility in a gauge theory with the SU(3) symmetry group is calculated by the Monte Carlo method. The result agrees with that calculated previously for the SU(2) group and is roughly two orders of magnitude smaller than that required for the solution assumed for the $U_A(1)$ problem.

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Monte Carlo calculations in lattice gauge theories are the most effective method for deriving nonperturbative quantities such as the tension in a string, the masses of glueballs and mesons, and the temperature of the deconfinement phase transition.[1] The vacuum expectation values of the constituent gluon and quark operators are of particular interest. On the one hand, calculations of these values can give an idea of the structure of the ground state; on the other, such calculations constitute an independent test for self-consistency of phenomenological approaches, e.g., the sum-rule approach of the Institute of Theoretical and Experimental Physics.[2]

In this paper we examine the "topological susceptibility" χ of the vacuum state in the Yang-Mills theory:

$$\chi \equiv -\left.\frac{d^2 P}{d\theta^2}\right|_{\theta=0} = \int d^4x \, \langle Q(x)\, Q(0)\rangle_{\text{no quarks}}, \tag{1}$$

where

$$Q(x) = \frac{g^2}{64\pi^2} G^a_{\mu\nu}\, \epsilon_{\mu\nu\rho\sigma}\, G^a_{\rho\sigma}(x)$$

is the density of the topological charge, and P and Q are respectively the pressure and phase of the vacuum.

The quantity χ plays a fundamental role in the solution of the $U_A(1)$ problem by means of Ward identities, which incorporate the anomaly of the $U_A(1)$ current and the existence of topologically nontrivial field configurations (instantons, etc.). Witten[3] has established the relationship between χ and the mass of the η' meson in the limit of a large number of colors. The effective-Lagrangian approach can refine this relation[4]:

$$\chi = \frac{1}{2N_l} F_\pi^2 (m^2_{\eta'} + m^2_\eta - m^2_{K^0} - m^2_{K^+}) \cong (182\ \text{MeV})^4, \tag{2}$$

where $N_l = 3$ is the number of light fermions, and $F_\pi \cong 95$ MeV is the pion decay constant.

The topological susceptibility was recently determined by Di Vecchia *et al.*[5] for the SU(2) gauge theory by Monte Carlo calculations using various definitions of the density of the topological charge on the lattice. Since their result, $\chi_{SU(2)} = (55 \pm 10\ \text{MeV})^4$, is two order of magnitude smaller than (2), we are interested in whether the relationship between the calculated and phenomenological values might be improved in the case of a realistic gauge theory with the SU(3) group.

Let us examine the definition of the density of topological charge:

$$Q_L(n) = -\frac{1}{2^9 \pi^2} \sum_{\mu\nu\rho\sigma = \pm 1}^{\pm 4} \tilde{\epsilon}_{\mu\nu\rho\sigma} \operatorname{tr}\left(U(n)_{\mu\nu} U(n)_{\rho\sigma}\right)$$

$$\xrightarrow[a \to 0]{} a^4 Q(x_n) + O(a^5), \tag{3}$$

where $\tilde{\epsilon}_{1234} = -\tilde{\epsilon}_{2134} = -\tilde{\epsilon}_{-1234} = \cdots = 1$, a is the lattice step, and $U(n)_{\mu\nu}$ is the matrix of a plaquet at node n in the μ–ν plane. We determine χ from "data" on the correlation function

$$a^4 \chi_L \equiv \sum_n \langle Q_L(n) Q_L(0) \rangle \tag{4}$$

obtained on lattices with 4^4 and 6^4 nodes with periodic boundary conditions. In the weak-coupling region, the behavior of χ_L is described by

$$\pi^4\, 2^{15}\, a^4 \chi_L = c_1 g_0^6 + c_2 g_0^8 + \ldots + \pi^4\, 2^{18}\, (\beta_0 g_0^2)^{-2\beta_1/\beta_0^2} \exp\left(-\frac{2}{\beta_0 g_0^2}\right) \frac{\chi}{\Lambda_L^4}, \tag{5}$$

where $\beta_0 = 11/16\pi^2$, $\beta_1 = 102/256\ \pi^4$, and $c_1 = 400.9$ (for a 4^4 lattice). At larger values of g_0 the calculated results reproduce the high-temperature expansion well:

$$\pi^4 \quad a^4 \chi_L = 768\left(1 + \frac{2}{3} g_0^{-2} + \frac{7}{3} g_0^{-4} + O(g_0^{-6})\right).$$

Calculations were carried out on an ES 1060 computer at the Joint Institute for Nuclear Research by the heat-sink algorithm[6] with a random choice of lattice ribs (random in the case of the 4^4 lattice). For each value of g_0 considered we took an average over 180 and 130 states for the 4^4 and 6^4 lattices, respectively.

In the scaling region $(0.9 \lesssim g_0^{-2})$ we found clear evidence for a nonperturbative contribution. The perturbative background can be described quite well by the low-order term $\mathcal{O}(g_0^6)$; a χ^2 fit of the next term yields a relatively small value, $c_2 = 42 \pm 20$ (curve A). From Eq. (5) we find the parameter in units of Λ $[\Lambda_L = (0.007) \pm (0.001)\sqrt{\sigma}$ (Ref. 7), where $\sqrt{\sigma} \cong 420$ MeV]:

$$\chi_{SU(3)} = (1.0 \pm 0.2) \cdot 10^5 \Lambda_L^4 \cong (52 \pm 8\ \text{MeV})^4 \tag{6}$$

(cf. curve B). The agreement with the value of $\chi_{SU(2)}$ confirms the suggestion $\chi_{SU(N_c)} = \mathcal{O}(N_c^0)$ if the string tension σ does not depend on N_c. There is an obvious discrepancy with the phenomenologically expected value, (2) (curve C). As expected, the reason for this discrepancy is not the fact that the lattice assumed here is small, as can be seen from the data obtained for the 6^4 lattice, precisely in the region where the discrepancy with the perturbation theory is observed.

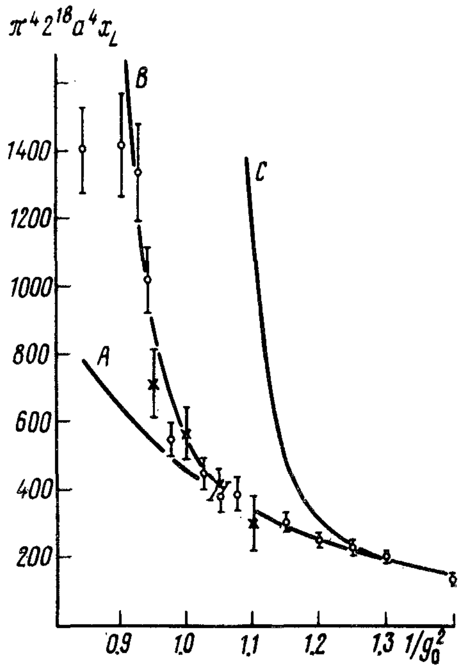

FIG. 1. Topological susceptibility for a gauge theory with the SU(3) group. Circles—4^4 lattice; crosses—6^4 lattice (the error bars show the standard deviation). The curves A, B, and C are plotted from Eq. (5) with the values $c_4 \cong 40$, $\chi = 0$; $1.0 \times 10^5 \Lambda_L^4$; and $1.5 \times 10^7 \Lambda_L^4 \cong (182\ \text{MeV})^4$, respectively.

We close with two comments. 1) Before we are convinced that the solution assumed for the $U_A(1)$ problem is not correct, we would like to test the value in (6) in the case of a topological-charge density which does not contain a nonperturbative background.[8] It should be noted, however, that our method has worked well in a corresponding determination of the gluon condensate.[9] 2) So far, the semiclassical approach (involving instantons) yields no indication of whether estimate (6) is correct (or incorrect). Our result is consistent with previous determinations of a lower limit[10] and an upper limit.[11]

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