# Drift solitons in a shallow rotating fluid

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Solitary cyclones and anticyclones and also a solitary cyclone-anticyclone pair, whose characteristic size is of the order of the Rossby-Obukhov length, are obtained in shallow water in a rotating paraboloidal vessel. Transfer of energy from small vortices to large vortices is observed visually.

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Observation of waves in water still gives new results and permits understanding complex phenomena in different areas of physics. Solitons-anticyclones in shallow rotating fluid, which have revealed a number of properties of drift waves in plasma and spots in Jupiter's atmosphere, and have enlarged the small class of experimentally observed non-one-dimensional solitons, are an example.[1,2]

It is shown in Refs. 1 and 3 that the equations of drift waves in a plasma and Rossby waves, after simplifying approximate transformations, lead to the same model equation, which has a stationary solution in the form of solitary two-dimensional solitons-anticyclones, which are larger than the characteristic dispersion length. It is also indicated therein that such solitons can arise in a shallow fluid in a rotating vessel with a nearly paraboloidal bottom profile. It was indeed possible to realize solitary anticyclones by this method.[2,4–6]

However, the dimensions of the vessel in these experiments were not sufficiently large to observe relatively smaller scale vortices.

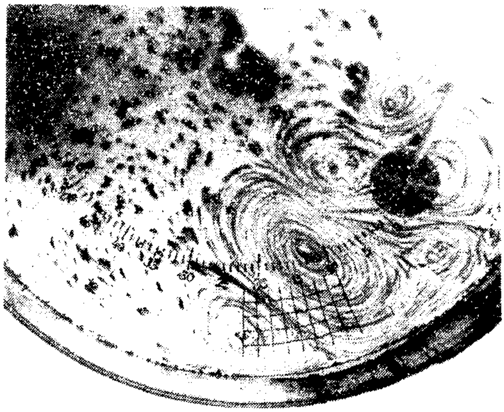
To obtain and investigate vortices, whose horizontal size is of the order of or less than the dispersion length (but much larger than the depth of the fluid), a setup was constructed at the Abastumani Astrophysical Observatory consisting of a rotating paraboloidal vessel whose dimensions (diameter 86 cm, radius of curvature at the top 92 cm) are approximately three times larger than the vessel used in Refs. 2 and 4–6. The period of rotation of the vessel (1.9 s) is such that in addition to photographing the vortices (with the help of a camera rotating together with the vessel) they can also be observed visually.

The methods used to excite the vortices and to measure the fluid velocity in them are analogous to those used in Ref. 2. The velocity of the fluid in a vortex is estimated from the exposure and length of the tracks of paper circles floating on the surface of the fluid, and the vortices are excited by briefly rotating a disk mounted at the bottom of the vessel.

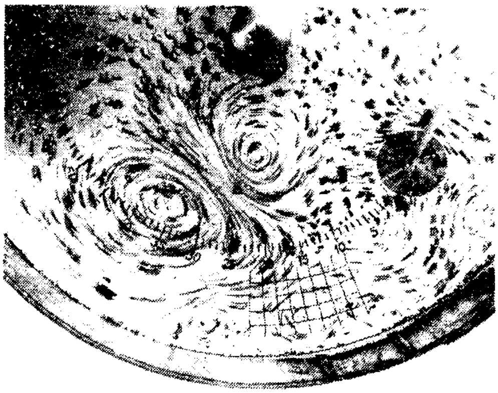
The experiments were conducted with different depths of fluid in the range 2–4 cm, when the depth gradient was small, because this gradient enters into the expression for the Rossby velocity $v_R$ (1) with a large coefficient, so that even a small error in measuring the gradient makes $v_R$ uncertain. A transparent cover was placed on the vessel in order to eliminate the effect of wind, which appreciably distorts the pattern. The effect of viscosity and surface tension was negligible due to the large dimensions, which is important for making comparisons with theory.

The indicated changes in the parameters of the system led to qualitatively new results. Because of the comparatively large depth of the fluid layer, it was possible to realize an equivalent cyclone-anticyclone pair (Fig. 1), predicted theoretically by Larichev and Reznik,[7] and a quite unexpected formation—a solitary cyclone (Fig. 2) (a vortex with a concave surface rotating in the same direction as the vessel and traveling, as does the anticyclone, in a direction opposite to the rotation of the vessel). The velocity of the cyclone when it moves in the parallel direction must exceed the Rossby velocity (1); however, it is difficult to determine the latter with sufficient accuracy. Solitary anticyclones (vortices with a convex surface, in which the fluid rotates opposite the direction of rotation of the vessel) are also easily excited. The velocity of the anticyclone whose motion is in the parallel direction (Fig. 3) is close to the velocity of the cyclones. An interesting phenomenon was observed experimentally: transfer of energy along scales in vortical turbulence, often studied in theoretical problems. In this case, numerous closely packed vortices were excited with the help of the disk (Fig. 1). As a result of their interaction, small vortices disappeared and a pair of comparatively large standing vortices formed, reminiscent of a cyclone-anticyclone solitary pair.

In a number of cases, it was observed that cyclones or anticyclones emitted waves



a



b

FIG. 1. a, b) Transfer of energy in vortical turbulence with the formation of a cyclone-anticyclone pair. The time between frames is 7.5 s. Here and in subsequent figures, the fluid depth is 2.5 cm, and the Rossby length coincides with the diameter of the exciting disk on the right side of the figure. Distances can be estimated according to the centimeter scale along the latitude at the bottom of the vessel.

and were gradually deflected toward the "north" or "south" from the initial latitude. According to theory, if the velocity of the vortex exceeds $v_R$, emission does not occur due to dispersive screening, since the velocity of small waves is less than $v_R$, and, for this reason, they cannot be in resonance with the vortex. This is evident from the dispersion equation for Rossby waves

$$\omega = \frac{k_x v_R}{1 + k^2 r_R^2}; \quad v_R = \frac{H_0 \omega_0 r}{2\mu R_0}\left(1 + \frac{\mu^3 R_0^2}{r H_0} \frac{\partial H_0}{\partial y}\right), \tag{1}$$

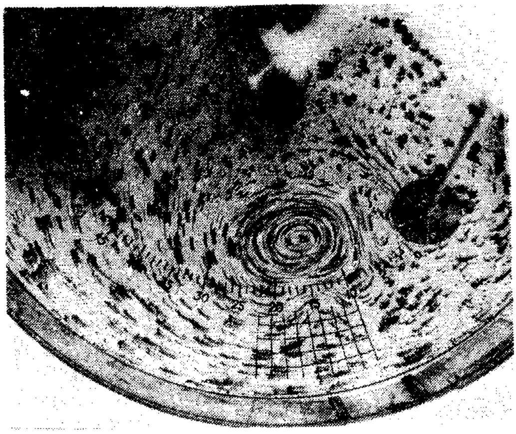


FIG. 2. Cyclone. The distance to the axis of the vessel is 30 cm and the translational velocity is 2.4 cm/s.

where **k** is the wave vector, $r_R^2 = H_0 R_0 \mu^3/4$, $r$ is the distance from the axis of the vessel to the center of the vortex, $R_0$ is the radius of curvature at the vertex of the paraboloid, $\omega_0$ is the rotational velocity of the vessel, $\mu^2 = 1 + (r/R_0)^2$ is the unperturbed depth, $\omega_0$ can depend on latitude $y$ oriented toward the edge of the vessel, and $x$ is the coordinate along the latitude.

It is evident from (1) that a packet of Rossby waves, when nonlinear effects are neglected, must spread out due to dispersion (wave number dependence of the group velocity). This spreading is especially large if $\Delta k r_R \gtrsim 1$, where $\Delta k$ is the spread of wave numbers in the packet, which occurs for vortices obtained here. The order of magnitude of the characteristic spreading time in this case, as is evident from (1), is

FIG. 3. Anticyclone. The translational velocity is 2.4 cm/s. The separatrix of the streamlines is visible.

$t_D \sim r_R/v_R$, which for vortices shown in the figures ($r_R = 9$ cm, $v_R = 2$ cm/s, $r = 30$ cm) amounts to 4 s. The lifetime of the observed vortices is 20–40 s, which is 5 to 10 times greater than $t_D$, so that we can assume that the dispersion spreading is canceled out by nonlinear self-compression, which is precisely characteristic of solitons. In Figs. 2–3, the rotational velocity of the fluid is maximum at a distance $\sim r_R$ from the center of the vortex and close to 5 cm/s, which is greater than the velocity of the vortex, since there is a region of trapped particles, first noted in Ref. 4, that moves together with the vortex. The amplitude of the perturbation of the surface can be estimated from the rotational velocity, starting from the balance between the pressure gradient and the Coriolis force. The ratio of $H_0$ to the perturbation of the surface by the vortex in our case is much greater than 5, which is confirmed by visual observation.

The angular rotational velocity of the fluid in the vortices is approximately 6 times smaller than $\omega_0$. Apparently, the paired solitons observed here in the form of a cyclone-anticyclone agree with the solution of the usual geostrophic equation, found in Ref. 7, which is valid for $kr_R \gtrsim 1$. In the opposite case, $k^2 r_R^2 \ll 1$, it was shown in Refs. 1 and 3 that only anticyclones are possible, which agrees with observations by M. V. Nezlin's group.[2,4–6] A cyclone-anticyclone pair was also observed in Ref. 5, but the velocity of rotation in the cyclone was "clearly smaller" than in the anticyclone and for this reason it was concluded that the cyclones are secondary. This nonequivalence can be explained by the intermediate value of the characteristic quantity $kr_R$.

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