Nonplanar contours and string tension in lattice gauge theories

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(Submitted 9 March 1983)
Pis'ma Zh. Eksp. Teor. Fiz. **37**, No. 11, 550–552 (5 June 1983)

An analog of an off-axis string is introduced in a Euclidean approach to gauge fields. A method for constructing a high-temperature expansion is described. The relationship between the diagrams in the Euclidean and Hamiltonian theories is established.

PACS numbers: 11.15.Ha, 11.10.Ef

The accuracy with which the string tension can be determined in lattice gauge theories worsens when we enter the roughening stage.

Kogut *et al.*[1] have analyzed the dynamics of a string between a quark and an antiquark which are not on a common axis. Working in a Hamiltonian gauge theory, they calculated the energy of the system in the first few orders of a high-temperature expansion. They showed that the string is in the roughening stage over the entire range of the coupling constant, $0 < g^2 < \infty$.

In this letter we examine an analogous problem for an off-axis string in a Euclidean approach to lattice gauge theories.

We consider the following process: At the time 0, a quark and an antiquark are created at a distance $\mathbf{R}$ from each other, and after a time t they disappear. If $\mathbf{R}$ does not lie on one of the lattice axes, there is a set of closed contours C_I of minimum area (the area of the surface that can cover the contour) which describe the trajectory of a quark and an antiquark. By analogy with Ref. 2, we can write the physical amplitude for this process as

$$A = \Sigma\, W(C_I),\tag{1}$$

where $W(C)$ is the Wilson order parameter. The potential energy of the system $V(\mathbf{R})$ and the string tension α are given by the following expresions:

$$V(\mathbf{R}) = -\lim_{t\to\infty} \ln A/t,\tag{2}$$

$$\alpha = \lim_{t\to\infty} V(R)/R.\tag{3}$$

In principle, the parameter $W(C)$ determined for any nonplanar contour can also serve as a source for determining α in the limit of large contours of this type. Since A is the physical amplitude, however, the tension α, which is determined from A, must be more regular (as a function of g^2). We would naturally expect that this circumstance would improve the accuracy of the determination of α.

Furthermore, by examining the amplitude A we can determine the Coulomb term in the potential energy of the string.

To show how to construct a high-temperature expansion in a Euclidean lattice gauge theory, we denote by $|I\rangle$ the Ith minimum path between the quark and the antiquark in a $(d-1)$-dimensional space. By definition we then have

$$A = \sum_I \langle I|T|I\rangle\tag{4}$$

where the summation is over those contours whose opposite sides are parallel.

We denote the eigenvectors and eigenvalues of the matrix T by $|J\rangle$ and $\exp(-E_J t)$; then we can write

$$V(\mathbf{R}) = -\lim_{t\to\infty} \frac{\ln\left(\sum_k \exp(-E_k t)\right)}{t} = E_1,\tag{5}$$

where $|1\rangle$ is the state with the largest value of $\exp(-Et)$. We wish to find an expansion of $V(\mathbf{R})$ as a series in the powers β^n ($\beta \equiv 1/g^2$). From (5) we have the condition $(E_2 - E_1)t \to \infty$, $\beta \to 0$. We cannot calculate $V(\mathbf{R})$ from $W(C_I)$ alone. If, on the other hand, we take $\langle 1'|T|1'\rangle$, where $|1'\rangle$ is the solution of the secular equation from Ref. 1, then we can express it in perturbation theory as an exponential function with an accuracy to $\beta^3 t^2$:

$$\langle 1'|T|1'\rangle = \beta^{\sqrt{2}Rt} \exp(-c_1\beta t)[\exp(c_2\beta + c_3\beta^2 + c_4\beta^2 t + O(\beta^3 t^2))].\tag{6}$$

According to (5), however, this means that we can find $V(\mathbf{R})$ to terms β^2 inclusively. We have numerically calculated n_I/R, where n_I is the number of surfaces covering the trajectory of the quark and antiquark with a common angular factor. For $\mathbf{R}$ at an angle of $\pi/4$ from the axes the calculations yield

$$V(R) = -(-1.414\ln u - 0.901\,u + 0.213\,u^2)R + (-0.181\,u + 0.003\,u^2)\frac{1}{R}.\tag{7}$$

Diagrams of the high-temperature expansion in the Hamiltonian gauge theory can be calculated on an anisotropic Euclidean lattice through a sampling summation of the infinite number of diagrams which are parallelepipeds stretched out along the t axis. For a string on one axis, for example, we have

$$\alpha = u_0^2 u_1^2 + u_0^2 u_1^4 + \ldots = \frac{u_0^2 u_1^2}{(1 - u_1^2)} , \tag{8}$$

where

$$u_0 = \int \exp[\beta_0 S(\Omega)] \chi_\phi(\Omega) d\Omega, \; u_1 = \int \exp[\beta_1 S(\Omega)] \chi_\phi(\Omega) d\Omega, \; \chi_\phi(\Omega)$$

is the nature of the fundamental representation, and $\beta_0 = 1/g_0^2$ corresponds to a plaquet parallel to the t axis and perpendicular to t. In the limit $g_0^2 \to \infty$, $g_1 g_0 \to g^2$, $g_1^2 \to 0$ we find the value of the specific diagram in the Hamiltonian theory:

$$\lim \frac{u_0{}^2 u_1^2}{1 - u_1^2} = \beta / C_2 , \tag{9}$$

where C_2 is the value of the second-order Casimir operator.

The relationship which has been found between the diagrams of Ref. 1 and those required for calculating A supports our choice of A.

I wish to thank S. G. Matinyan, A. A. Migdal, A. M. Polyakov, and A. G. Sedrakyan for useful discussions.

[1]J. B. Kogut *et al.*, Phys. Rev. D **23**, 2945 (1981).
[2]K. G. Wilson, Phys. Rev. D **10**, 2445 (1974).

Translated by Dave Parsons
Edited by S. J. Amoretty