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Spectroscopy of baryons containing two heavy quarks in nonperturbative quark dynamics

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We have studied the three quark systems in an Effective Hamiltonian approach in QCD. With only two parameters: the string tension $\sigma = 0.15\,\mathrm{GeV}^2$ and the strong coupling constant $\alpha_s = 0.39$ we obtain a good description of the ground state light and heavy baryons. The prediction of masses of the doubly heavy baryons not discovered yet are also given. In particular, a mass of 3637 MeV for the lightest ccu baryon is found by employing the hyperspherical formalism to the three quark confining potential with the string junction.

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The discovery of the B_c meson [1] demonstrates that new sectors of hadron physics are becoming accessible to experiment. In particular, the existence of doubly heavy baryons is a natural consequence of the quark model, and it would be surprising if they did not exist. Data from the BaBar and Belle collaborations at the SLAC and KEK B-factories would be good places to look for doubly charmed baryons. Recently the SELEX, the charm hadroproduction experiment at Fermilab, reported a narrow state at $3519 \pm 1\,\mathrm{MeV}$ decaying in $\Lambda_c^+ K^- \pi^+$, consistent with the weak decay of the doubly charged baryon Ξ_{cc}^+ [2]. The SELEX result was recently critically discussed in [3]. Whether or not the state that SELEX reports turns out to be the first observation of doubly charmed baryons, studying their properties is important for a full understanding of the strong interaction between quarks.

Estimations for the masses and spectra of the baryons containing two or more heavy quarks have been considered by many authors [4]. The purpose of this letter is to present a consistent treatment of the masses and wave functions of the light, heavy and doubly heavy baryons obtained in a simple approximation within the nonperturbative QCD. In Ref.[5] starting from the QCD Lagrangian and assuming the minimal area for the asymptotics of the Wilson loop the Hamiltonian of the $3q$ system in the rest frame has been derived. The methodology of the approach has been reviewed recently [6]. By using this approach and the hypercentral approximation [7] we calculate the ground state energies and wave functions of the doubly heavy baryons as three quark systems, with the three-body confinement force. As a by-product, we also report the masses and wave functions for light and heavy baryons.

From experimental point of view, a detailed discussion of the excited QQ'q states is probably premature. Therefore we consider the ground state baryons without radial and orbital excitations in which case tensor and spin-orbit forces do not contribute perturbatively. Then only the spin-spin interaction survives in the perturbative approximation. The Effective Hamiltonian (EH) has the following form [6]

$$H = \sum_{i=1}^{3} \left(\frac{m_i^{(0)2}}{2m_i} + \frac{m_i}{2} \right) + H_0 + V, \qquad (1)$$

where H_0 is the non-relativistic kinetic energy operator, V is the sum of the perturbative one-gluon exchange potentials V_c:

$$V_c = -\frac{2}{3}\alpha_s \cdot \sum_{i<j} \frac{1}{r_{ij}}, \qquad (2)$$

and the string potential

$$V_{\text{string}}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \sigma l_{\min}, \qquad (3)$$

where $l_{\min}$ is the sum of the three distances $|\mathbf{r}_i|$ from the string junction point. In contrast to the standard approach of the constituent quark model the dynamical masses m_i are no longer free parameters. They are expressed in terms of the running masses $m_i^{(0)}(Q^2)$ defined at the appropriate hadronic scale of Q^2 from the condition of the minimum of the baryon mass $M_B^{(0)}$ as function of m_i:

$$\frac{\partial M_B^{(0)}(m_i)}{\partial m_i} = 0,$$
$$M_B^{(0)} = \sum_{i=1}^{3} \left(\frac{m_i^{(0)2}}{2m_i} + \frac{m_i}{2} \right) + E_0(m_1, m_2, m_3), \qquad (4)$$

E_0 being eigenvalue of the operator $H_0 + V$. Technically, this has been done using the einbein (auxiliary fields) approach, which is proven to be rather accurate in various calculations for relativistic systems. Einbeins are treated as c number variational parameters: the eigenvalues of the EH are minimized with respect to einbeins to obtain the physical spectrum. Such procedure provides the reasonable accuracy for the meson ground states [8].

The physical mass M_B of a baryon is [9]

$$M_B = M_B^{(0)} + C, \quad C = -\frac{2\sigma}{\pi}\sum_i \frac{\eta_i}{m_i}, \tag{5}$$

where the constant C has the meaning of the quark self energy. The values of η_i are taken from [9]. They are 1, 0.88, 0.234, and 0.052 for q, s, c, and b quarks, respectively.

The EH is solved using the hyperspherical approach adequate for confining potentials. The baryon wave function depends on the three-body Jacobi coordinates

$$\boldsymbol{\rho}_{ij} = \sqrt{\frac{\mu_{ij}}{\mu}}(\mathbf{r}_i - \mathbf{r}_j), \quad \boldsymbol{\lambda}_{ij} = \sqrt{\frac{\mu_{ij,k}}{\mu}}\left(\frac{m_i\mathbf{r}_i + m_j\mathbf{r}_j}{m_i+m_j} - \mathbf{r}_k\right) \tag{6}$$

$(i,j,k$ cyclic), where μ_{ij} and $\mu_{ij,k}$ are the appropriate reduced masses

$$\mu_{ij} = \frac{m_i m_j}{m_i+m_j}, \quad \mu_{ij,k} = \frac{(m_i+m_j)m_k}{m_i+m_j+m_k}, \tag{7}$$

and μ is an arbitrary parameter with the dimension of mass which drops off in the final expressions. In terms of the Jacobi coordinates the kinetic energy operator H_0 is written as

$$H_0 = -\frac{1}{2\mu}\left(\frac{\partial^2}{\partial\boldsymbol{\rho}^2} + \frac{\partial^2}{\partial\boldsymbol{\lambda}^2}\right) = -\frac{1}{2\mu}\left(\frac{\partial^2}{\partial R^2} + \frac{5}{R}\frac{\partial}{\partial R} + \frac{K^2(\Omega)}{R^2}\right), \tag{8}$$

where R is the six-dimensional hyper-radius $R^2 = \boldsymbol{\rho}_{ij}^2 + \boldsymbol{\lambda}_{ij}^2$, and $K^2(\Omega)$ is angular momentum operator whose eigen functions (the hyper spherical harmonics) are $K^2(\Omega)Y_{[K]} = -K(K+4)Y_{[K]}$, with K being the grand orbital momentum. In terms of $Y_{[K]}$ the wave function $\psi(\boldsymbol{\rho},\boldsymbol{\lambda})$ can be written in a symbolical shorthand as [10]

$$\psi(\boldsymbol{\rho},\boldsymbol{\lambda}) = \sum_K \psi_K(R)Y_{[K]}(\Omega). \tag{9}$$

In the hyper radial approximation which we shall use below $K = 0$ and $\psi = \psi(R)$. Since R^2 is exchange symmetric the baryon wave function is totally symmetric under exchange. Introducing the variable $x = \sqrt{\mu}R$ and averaging the interaction $U = V_c + V_{\text{string}}$ over the six-dimensional sphere Ω_6 one obtains the Schrödinger equation for $u(x) = x^{5/2}\psi(x)$:

$$\frac{d^2u(x)}{dx^2} + 2\left[E_0 + \frac{a}{x} - bx - \frac{15}{8x^2}\right]u(x) = 0, \tag{10}$$

with the boundary conditions $u(x) \sim \mathcal{O}(x^{5/2})$ as $x \to 0$ and the asymptotic $u(x) \sim Ai(y) \sim \frac{1}{2}\pi^{-1/2}y^{-1/4}\exp(-\frac{2}{3}y^{3/2})$, $y = (2b)^{1/3}x$, as $x \to \infty$. In Eq.(10) E_0 is the ground state eigenvalue and

$$a = \frac{2\alpha_s}{3}\cdot\frac{16}{3\pi}\cdot\sum_{i<j}\sqrt{\mu_{ij}}, \quad b = \frac{1}{R\sqrt{\mu}}\int V_{\text{string}}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)\cdot\frac{d\Omega_6}{\pi^3}. \tag{11}$$

The potential $V_{\text{string}}(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3)$ has rather complicated structure. In the Y-shape, the string meet at 120° in order to insure the minimum energy. This shape moves continuously to a two–legs configuration where the legs meet at an angle larger than 120°. Let φ_{ijk} be the angle between the line from quark i to quark j and that from quark j to quark k. If φ_{ijk} are all smaller than 120°, then the equilibrium junction position coincides with the so-called Torichelli point of the triangle in which vertices three quarks are situated. In this case, in terms of the variables x, $\theta = \arctan(\rho_{12}/\lambda_{12})$, and $\cos\chi = \boldsymbol{\rho_{12}}\cdot\boldsymbol{\lambda_{12}}/\rho_{12}\lambda_{12}$ $(0 \leqq \theta \leqq \pi/2,\ 0 \leqq \chi \leqq \pi)$ one obtains

$$l^2_{\min} = x^2\cos^2\theta\left(\frac{(m_1^3 - m_2^3)\tan^2\theta}{m_1 m_2(m_1^2 - m_2^2)} + \left(\frac{m_2 - m_1}{m_2 + m_1}\cos\chi + \sqrt{3}\sin\chi\right)\frac{\tan\theta}{m} + \frac{1}{\mu_{12,3}}\right), \tag{12}$$

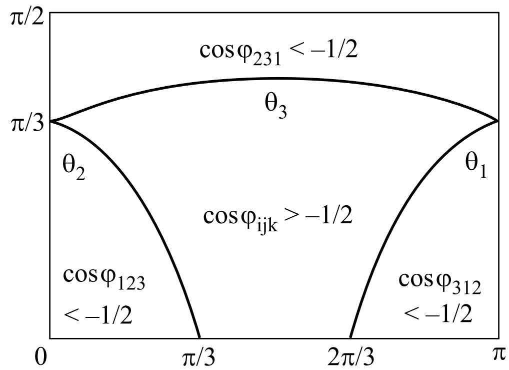
where $m^2 = m_1m_2m_3/(m_1+m_2+m_3)$. For the case $m_1 = m_2 = m_3$ this expression coincides with that derived in Ref.[11]. If $\varphi_{ijk} > 120°$, the lowest energy configuration has the junction at the position of quark j and $l_{\min} = r_{ij} + r_{jk}$, where

$$r_{12} = \frac{x\sin\theta}{\sqrt{\mu_{12}}}, \quad r_{13} = \frac{x\cos\theta}{\sqrt{\mu_{12,3}}}\sqrt{\frac{m^2}{m_1^2}\tan^2\theta + \frac{2m}{m_1}\tan\theta\cos\chi + 1}, \tag{13}$$

and

$$r_{23} = \frac{x\cos\theta}{\sqrt{\mu_{12,3}}}\sqrt{\frac{m^2}{m_2^2}\tan^2\theta - \frac{2m}{m_2}\tan\theta\cos\chi + 1}. \tag{14}$$

The boundaries corresponding to the various regions in the (θ,χ) plane are: $\theta_{1(2)}(\chi) = \arctan(m_{1(2)}(\mp\cos\chi - \sin\chi/\sqrt{3})/m)$, $\theta_3(\chi) = \arctan(m_2(f(\chi) + \sqrt{f^2(\chi)+4\kappa})/2m)$, $f(\chi) = (1-\kappa)\cos\chi + (1+\kappa)\sin\chi/\sqrt{3}$, $\kappa = m_1/m_2$. These boundaries are shown in Figure for the case of equal quark masses.



The four regions in the (θ,χ) plane corresponding to $\varphi_{ijk} \geq 120^\circ$ and $\varphi_{ijk} \leq 120^\circ$ for the case of equal quark masses

Note that the frequently used approximation [12, 6] is to choose the string junction point as coinciding with the center-of-mass coordinate. In this case

$$V_{\text{string}} = \sigma \sum_{i<j} \frac{1}{m_k} \cdot \sqrt{\mu\mu_{ij,k}} \cdot |\boldsymbol{\lambda}_{ij}|,$$
$$b = \sigma \cdot \frac{32}{15\pi} \cdot \sum_{i<j} \frac{\sqrt{\mu_{ij,k}}}{m_k}. \qquad (15)$$

This approximation that greatly simplifies the calculations increases the value of b in Eq.(10) by $\sim 5\%$ as illustrated by the results of Table 1.

We first solve Eq.(4) for the dynamical quark masses m_i retaining only the string potential in the effective Hamiltonian (1). This procedure is consistent with Ref.[6], but different from that of [12]. Then we add the perturbative Coulomb potential and solve Eq.(10) to obtain the ground state eigenvalues E_0. The baryon masses M_B are then obtained from solving Eq.(5).

We use the same parameters as in Ref. [13]: $\sigma = 0.15$ GeV^2 (this value has been confirmed in a recent lattice study [14]), $\alpha_s = 0.39$, $m_q^{(0)} = 0.009$ GeV, $m_s^{(0)} = 0.17$ GeV, $m_c^{(0)} = 1.4$ GeV, and $m_b^{(0)} = 4.8$ GeV. In Table 2 for various three-quark states, we give the quark masses m_i, the ground state eigenvalues E_0, and the baryon masses M_B. For completeness, in the last column we report the values of the integral

$$\gamma = \int \frac{u^2(x)}{x^3} dx \qquad (16)$$

Таблица 1

Illustration of the accuracy of the approximation (15)

Baryon	Eq.(11)	Eq.(15)
qqq	1.583	1.663
qqs	1.556	1.636
qss	1.530	1.608
qsc	1.339	1.417
qqb	1.293	1.384
qcb	1.038	1.101
qbb	0.925	0.975

Shown are the values $b\sqrt{m_q}/\sigma$ given by Eqs.(11) and (15) where m_q is the lightest quark mass.

in terms of which the quantities $R_{ijk} = (4\mu_{ij}^{3/2}/\pi^2)\cdot\gamma$ are expressed determining the probability to find a quark i at the location of the quark j in a baryon ijk. These quantities are of special importance for the lifetime calculations of heavy hadrons.

Таблица 2

Baryon	m_1	m_2	m_3	E_0	M_B	γ
qqq	0.362	0.362	0.362	1.392	1.144	0.1389
qqs	0.367	0.367	0.407	1.362	1.242	0.1369
qss	0.371	0.411	0.411	1.335	1.336	0.1351
sss	0.415	0.415	0.415	1.307	1.426	0.1333
qqc	0.406	0.406	1.470	1.142	2.464	0.1241
qsc	0.409	0.448	1.471	1.116	2.542	0.1228
ssc	0.452	0.452	1.473	1.090	2.621	0.1214
qqb	0.425	0.425	4.825	1.054	5.823	0.1201
qsb	0.429	0.469	4.826	1.026	5.903	0.1188
ssb	0.471	0.471	4.826	1.000	5.975	0.1177
qcc	0.444	1.494	1.494	0.876	3.659	0.1143
scc	0.485	1.496	1.496	0.851	3.726	0.1134
qcb	0.465	1.512	4.836	0.753	6.969	0.1136
scb	0.505	1.514	4.837	0.729	7.032	0.1128
qbb	0.488	4.847	4.847	0.567	10.214	0.1207
sbb	0.526	4.851	4.851	0.544	10.273	0.1202

For various $3q$ systems in column (1) we display the dynamical quark masses given by Eq.(4), the ground state eigenvalue E_0 in Eq.(10), the baryon masses including the self energy correction Eq.(5) (all in units of GeV) and the correlation function γ, Eq.(16) (in units GeV$^{3/2}$)

Note that there is no good theoretical reason why quark masses m_i need to be the same in different baryons. Inspection of Table 2 shows that the masses of the light quarks (u, d or s) are increased by ~ 100 MeV when going from the light to heavy baryons. The dynamical masses of light quarks $m_q \sim \sqrt{\sigma} \sim 400-500$ MeV qualitatively agree with the results of

Ref.[13] obtained from the analysis of the heavy–light ground state mesons.

While studying Table 2 is sufficient to have an appreciation of the accuracy of our predictions, few comments should be added. We expect an accuracy of the baryon predictions to be $\sim 5-10\%$ that is partly due to the approximations employed in the derivations of the EH itself [6] and partly due to the error associated with the variational nature of hyperspherical approximation. From this point of view the overall agreement with data is quite satisfactory. For example we get $\frac{1}{2}(N+\Delta)_{\rm theory} = 1144$ MeV vs $\frac{1}{2}(N+\Delta)_{\rm exp} = 1085$ MeV (a 5% increase in α_s would correctly give the $N-\Delta$ center of gravity), $\frac{1}{4}(\Lambda+\Sigma+2\Sigma^*) = 1242$ MeV vs experimental value of 1267 MeV. We also find $\Xi_{\rm theory} = 1336$ MeV (without hyperfine splitting) vs $\Xi^{1/2}_{\rm exp} = 1315$ MeV and $\Xi_{c\rm theory} = 2542$ Mev vs $\Xi_{c\,\rm exp} = 2584$ MeV. On the other hand, our study shows some difficulties in reproducing e.g. the Ω-hyperon mass.

Таблица 3

Comparison of our predictions for ground state masses (in units of GeV) of doubly heavy baryons with other predictions

Baryon	This Work	Ref.[12]	Ref.[15]	Ref.[16]	Ref.[17]
Ξ_{cc}	3.66	3.69	3.57	3.69	3.70
Ω_{cc}	3.73	3.86	3.66	3.84	3.80
Ξ_{cb}	6.97	6.96	6.87	6.96	6.99
Ω_{cb}	7.03	7.13	6.96	7.15	7.07
Ξ_{bb}	10.21	10.16	10.12	10.23	10.24
Ω_{bb}	10.27	10.34	10.19	10.38	10.34

In Table 3 we compare the spin–averaged masses (computed without the spin–spin term) of the lowest double heavy baryons to the predictions of other models [15–17] as well as variational calculations of Ref.[12] for which the center of gravity of non-strange baryons and hyperons is essential a free parameter. Most of recent predictions were obtained in a light quark-heavy diquark model [15, 16], in which case the spin-averaged values are $M = \frac{1}{3}(M_{1/2} + 2M_{3/2})$. Note that the wave function calculated in the hyperspherical approximation shows the marginal diquark clustering in the doubly heavy baryons. This is principally kinematic effect related to the fact that in this approximation the difference between the various mean values $\bar{r}_{ij}$ in a baryon is due to the factor $\sqrt{1/\mu_{ij}}$ which varies between $\sqrt{2/m_i}$ for $m_i = m_j$ and $\sqrt{1/m_i}$ for $m_i \ll m_j$. In general, in spite of the completely different physical picture, we find a reasonable agreement within 100 MeV between different predictions for the ground state masses of the doubly heavy baryons. Our prediction for M_{ccu} is 3.66 GeV with the perturbative hyperfine splitting $\Xi^*_{ccu} - \Xi_{ccu} \sim 40$ MeV. Note that the mass of Ξ^+_{cc} is rather sensitive to the value of the running c-quark mass $m_c^{(0)}$ [18].

In conclusion, we have have shown that baryon spectroscopy can be unified in a single framework of the Effective Hamiltonian which is consisted with QCD. This picture uses the stringlike picture of confinement and perturbative one-gluon exchange potential. The main advantage of this work is demonstration of the fact that it is possible to describe all the baryons in terms of the only two parameters inherent to QCD, namely σ and α_s.

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