Pis'ma v ZhETF, vol. 77, iss. 2, pp. 104 – 108 © 2003 January 25

Behavior of Fermi systems approaching fermion condensation quantum phase transition from disordered phase

V. R. Shaginyan[1)]

Petersburg Nuclear Physics Institute RAS, 188300 Gatchina, Russia

CTSPS, Clark Atlanta University, Atlanta, 30314 Georgia, USA

Submitted 23 December 2002

The behavior of Fermi systems which approach the fermion condensation quantum phase transition (FCQPT) from the disordered phase is considered. We show that the quasiparticle effective mass M^* diverges as $M^* \propto 1/|x - x_{FC}|$ where x is the system density and x_{FC} is the critical point at which FCQPT occurs. Such a behavior is of general form and takes place in both three dimensional (3D) systems and two dimensional (2D) ones. Since the effective mass M^* is finite, the system exhibits the Landau Fermi liquid behavior. At $|x - x_{FC}|/x_{FC} \ll 1$, the behavior can be viewed as a highly correlated one, because the effective mass is large and strongly depends on the density. In case of electronic systems the Wiedemann-Franz law is held and Kadowaki-Woods ratio is preserved. Beyond the region $|x - x_{FC}|/x_{FC} \ll 1$, the effective mass is approximately constant and the system becomes conventional Landau Fermi liquid.

PACS: 71.10.Hf, 71.27.+a, 74.72.–h

It is widely believed that unusual properties of the strongly correlated liquids observed in the high-temperature superconductors, heavy-fermion metals, 2D ^{3}He and etc., are determined by quantum phase transitions. Any quantum phase transition occurs at temperature $T = 0$ and is being driven by a control parameter other then temperature, for instance, by pressure, by magnetic field, or by the density x. A quantum phase transition occurs at the quantum critical point. As any phase transition, the quantum phase transition is related to the order parameter which induces a broken symmetry. Therefore, direct experimental studies of relevant quantum phase transitions are of crucial importance for understanding the physics of the high-temperature superconductivity and strongly correlated systems.

In case of the high-temperature superconductors, these experiments are difficult to carry out, because at low temperatures all the corresponding area is occupied by the superconductivity. On the other hand, experimental data on the behavior of different Fermi liquids, when systems are approaching the critical point from the disordered phase, can help to illuminate both the nature of this point and the nature of control parameter by which this phase transition is driven. Experimental facts on high-density 2D ^{3}He [1, 2] show that the effective mass diverges when the density at which 2D ^{3}He liquid begins to solidify is approached [2]. Then, the sharp increase of the effective mass in a metallic 2D electron system is observed when the density tends to the critical density of the metal-insulator transition point. This transition occurs at sufficiently low densities [3]. Note, that there is no ferromagnetic instability in both Fermi systems and the relevant Landau amplitude $F_0^a > -1$ [2, 3], in accordance with the almost localized fermion model [4].

Recent measurements for non-superconducting $La_{1.7}Sr_{0.3}CuO_4$ have shown that the resistivity ρ exhibits T^2 behavior, $\rho = \rho_0 + \Delta\rho$ with $\Delta\rho = AT^2$, that the Wiedemann-Franz (WF) law is verified to hold perfectly, and that the Kadowaki-Woods ratio, A/γ_0^2 [5], is enhanced compared with heavy-fermion metals [6]. Here γ_0 is the linear specific heat coefficient, $C = \gamma_0 T$. These data demonstrate the behavior of the Fermi liquid located above the critical point, or on the side of the disordered phase.

In this Letter, we study the behavior of Fermi systems which approach the fermion condensation quantum phase transition (FCQPT) [7] from disordered phase and show that the outlined experimental data can be explained within the framework of our approach. We analyze the appearance of FCQPT in different 2D and 3D Fermi liquids and show that at $T \to 0$ FCQPT manifests itself in the divergence of the quasiparticle effective mass M^* as the density x of a system approaches the critical point x_{FC} at which FCQPT takes place, so that $M^* \propto 1/|x - x_{FC}|$. Since the effective mass M^* is finite, the system exhibits the Landau Fermi liquid (LFL) behavior at low temperatures. At sufficiently high temperatures, the system possesses the non Fermi liquid

[1)] e-mail: vrshag@thd.pnpi.spb.ru

(NFL) behavior. At $|x - x_{FC}|/x_{FC} \ll 1$, this behavior can be viewed as a highly correlated one, because the effective mass strongly depends on the density and is large. We show that in the case of electronic systems the WF law is held, and Kadowaki-Woods ratio is preserved. Beyond the region $|x - x_{FC}|/x_{FC} \ll 1$, the effective mass is approximately constant and the system becomes conventional Landau Fermi liquid.

A new state of Fermi liquid with the fermion condensate (FC) [8, 9] which takes place beyond the critical point x_{FC} is defined by the equation [8]

$$\frac{E[n(p)]}{\delta n(\mathbf{p})} - \mu = \varepsilon(\mathbf{p}) - \mu = 0, \quad \text{if } 0 < n(\mathbf{p}) < 1;\ p_i \le p \le p_f \in L_{FC}. \tag{1}$$

Here $E[n(p)]$ is the Landau functional of the ground state energy, $n(\mathbf{p})$ is the quasiparticle distribution function, $\varepsilon(\mathbf{p})$ is the single-particle energy of the quasiparticles, and μ is the chemical potential [10]. At $T = 0$, Eq.(1) defines a new state of Fermi liquid with the FC for which the modulus of the superconducting order parameter $|\kappa(\mathbf{p})|$ has finite values in L_{FC} range of momenta $p_i \le p \le p_f$ occupied by FC. At the same time the superconducting gap can be infinitely small, $\Delta_1 \to 0$ in L_{FC} provided the corresponding pairing interaction is also small [7, 8]. Such a state can be considered as superconducting, with infinitely small value of Δ_1 so that the entropy of this state is equal to zero. This state, created by the quantum phase transition, disappears at $T > 0$. FCQPT can be considered as a "pure" quantum phase transition because it cannot take place at finite temperatures. Therefore, the corresponding quantum critical point does not represent the end of a line of continuous phase transitions at $T = 0$. Nonetheless, FCQPT has a strong impact on the system's properties up to temperature T_f above which FC effects become insignificant [7, 8]. FCQPT does not violate any rotational symmetry or translational symmetry, being characterized by the order parameter $\kappa(\mathbf{p}) = \sqrt{n(\mathbf{p})(1 - n(\mathbf{p}))}$. It follows from Eq. (1) that the quasiparticle system splits on the two quasiparticle subsystems: the first one in L_{FC} range is occupied by the quasiparticles with the effective mass $M^*_{FC} \propto 1/\Delta_1$, while the second by quasiparticles with finite mass M^*_L and momenta $p < p_i$. Note, that the existence of such a state can be revealed experimentally. Since the order parameter $\kappa(\mathbf{p})$ is suppressed by a magnetic field B, when $B^2 \sim \Delta_1^2$, a weak magnetic field B will destroy the state with FC converting the strongly correlated Fermi liquid into the normal Landau Fermi liquid.

Equation (1), possessing solutions at some density $x = x_{FC}$, determines the critical point of FCQPT. It is also evident from Eq. (1) that the effective mass diverges when $x \to x_{FC}$, $M^*_L(x \to x_{FC}) \to \infty$. Let us assume that FC has just taken place, $p_i \to p_f \to p_F$, and the deviation $\delta n(p)$ is small. Expanding functional $E[n(p)]$ in Taylor's series with respect to $\delta n(p)$ and retaining the leading terms, one obtains,

$$\mu = \varepsilon(\mathbf{p}, \sigma) = \varepsilon_0(\mathbf{p}, \sigma) + \sum_{\sigma_1} \int F_L(\mathbf{p}, \mathbf{p}_1, \sigma, \sigma_1) \delta n(\mathbf{p_1}, \sigma_1) \frac{d\mathbf{p}_1}{(2\pi)^2}; \quad p_i \le p \le p_f \in L_{FC}. \tag{2}$$

In Eq.(2) $F_L(\mathbf{p}, \mathbf{p}_1, \sigma, \sigma_1) = \delta^2 E / \delta n(\mathbf{p}, \sigma) \delta n(\mathbf{p}_1, \sigma_1)$ is the Landau interaction [10], and σ denotes the spin states. Both the Landau interaction and the single-particle energy $\varepsilon_0(p)$ are calculated at $n(p) = n_F(p)$. Here $n_F(p) = \theta(p_F - p)$, and $\theta(p - p_F)$ is the Fermi-Dirac distribution at $T = 0$. Equation (2) possesses solutions when the Landau amplitude F_L is positive and sufficiently large, so that the integral on the right hand side of Eq.(2) defining the potential energy is large and therefore the potential energy prevails over the kinetic energy $\varepsilon_0(\mathbf{p})$ [8]. At temperatures $T \ge T_c$, the effective mass M^*_{FC} related to FC is given by [7, 11],

$$M^*_{FC} \simeq p_F \frac{p_f - p_i}{4T}. \tag{3}$$

Multiplying both sides of Eq.(3) by $p_f - p_i$ we obtain the energy scale E_0 separating the slow dispersing low energy part related to the effective mass M^*_{FC}, from the faster dispersing relatively high energy part defined by the effective mass M^*_L [7, 11]

$$E_0 \simeq 4T. \tag{4}$$

It is clear from Eq.(4) that the scale E_0 does not depend on the range $p_f - p_i$. It is natural to assume that we have returned back to the Landau theory by integrating out high energy degrees of freedom and introducing the quasiparticles. Sole difference between the Landau Fermi liquid and Fermi liquid undergone FCQPT is that we have to expand the number of relevant low energy degrees of freedom by introducing new type of quasiparticles with the effective mass M^*_{FC} given by Eq.(3) and the energy scale E_0 given by Eq.(4). It is seen from Eqs. (1) and (2) that the FC quasiparticles form a collective state, since their energies are defined by the macroscopical number of quasiparticles within the region $p_i - p_f$. The shape of the spectra is not affected by the Landau interaction, which, generally speaking, depends on the system's properties, including the collective states, impurities, etc. The only thing defined by the interaction is

the width of the region $p_i - p_f$, provided the interaction is sufficiently strong to produce the FC phase transition at all. The spectra related to FC are of universal form and determined by T, as it follows from Eq.(3). Thus, the system's properties and dynamics are dominated by a strong collective effect originated from FCQPT and determined by the macroscopic number of quasiparticles in the range L_{FC}. Such a system can viewed as a strongly correlated system and cannot be disturbed by the scattering of individual quasiparticles, thermal excitations, impurities, and etc, and has features of a quantum protectorate [7, 12, 13].

The appearance of FCQPT in Fermi liquids, when the effective interaction becomes sufficiently large, has been predicted in Ref.[14]. FCQPT precedes the formation of charge-density waves or stripes, which take place at some value $x = x_{cdw}$ with $x_{FC} > x_{cdw}$, while the Wigner solidification takes place even at lower values of x and leads to an insulator. In the same way, the effective mass inevitably diverges as soon as the density x becomes sufficiently large approaching the critical density at which 2D ^{3}He begins to solidify, as it was observed in [2].

Now we consider the divergence of the effective mass in 2D and 3D Fermi liquids at $T = 0$, when the density x approaches FCQPT from the side of the normal LFL. First, we calculate the divergence of M^* as a function of the difference $(x_{FC} - x)$ in case of 2D ^{3}He. For this purpose we use the equation for M^* obtained in Ref.[14] where the divergence of effective mass M^* due to the onset of FC in different Fermi liquids including ^{3}He was predicted

$$\frac{1}{M^*} = \frac{1}{M} + \frac{1}{4\pi^2}[14].$$

$$\int_{-1}^{1}\int_{0}^{g_0} \frac{v(q(y))}{[1 - R(q(y), \omega = 0, g)\chi_0(q(y), \omega = 0)]^2} \frac{ydydg}{\sqrt{1-y^2}}. \quad (5)$$

Here we adopt the shorthand, $p_F\sqrt{2(1-y)} = q(y)$, with $q(y)$ is the transferred momentum, M is the bare mass, ω is the frequency, $v(q)$ is the bare interaction, and the integral is taken over the coupling constant g from zero to its real value g_0. In Eq.(5), both $\chi_0(q,\omega)$ and $R(q,\omega)$, being the linear response function of noninteracting Fermi liquid and the effective interaction respectively, define the linear response function of the system under consideration

$$\chi(q,\omega,g) = \frac{\chi_0(q,\omega)}{1 - R(q,\omega,g)\chi_0(q,\omega)}. \quad (6)$$

In the vicinity of charge density wave instability, occurring at the density x_{cdw}, the singular part of the function χ^{-1} on the disordered side is of the well-known form, see. e.g. [15]

$$\chi^{-1}(q,\omega,g) \propto (x_{cdw} - x) + (q - q_c)^2 + (g_0 - g), \quad (7)$$

where $q_c \sim 2p_F$ is the wavenumber of the charge density wave. Upon substituting Eq.(7) into Eq.(5) and performing the integrations, the equation for the effective mass M^* can be cast into the following form [16]

$$\frac{1}{M^*} = \frac{1}{M} - \frac{C}{\sqrt{x_{cdw} - x}}, \quad (8)$$

with C being some positive constant. It is seen form Eq. (8) that M^* diverges at some point x_{FC}, which is referred to as the critical point, as a function of the difference $(x_{FC} - x)$

$$M^* \sim M\frac{x_{FC}}{x_{FC} - x}. \quad (9)$$

It follows from the derivation of Eqs.(8) and (9) that the form of these equations is independent from the bare interaction $v(q)$, therefore both of these equations are also applicable to 2D electron liquid or to another Fermi liquid. It is also seen from Eqs.(8) and (9) that FCQPT precedes the formation of charge-density waves. As consequence of this, the effective mass diverges at high densities in case of 2D ^{3}He, and it diverges at low density in case of 2D electron systems, in accordance with experimental facts [2, 3]. Note, that in the both cases the difference $(x_{FC} - x)$ has to be positive because x_{FC} represents the solution of Eq.(8). Thus, considering electron systems we have to replace $(x_{FC} - x)$ by $(x - x_{FC})$. In case of 3D system, the effective mass is given by [14]

$$\frac{1}{M^*} = \frac{1}{M} + \frac{p_F}{4\pi^2} \times$$

$$\times \int_{-1}^{1}\int_{0}^{g_0} \frac{v(q(y))ydydg}{[1 - R(q(y), \omega = 0, g)\chi_0(q(y), \omega = 0)]^2}. \quad (10)$$

A comparison of Eq.(10) and Eq.(5) shows that there is no fundamental difference between these equations, and along the same lines we again arrive to Eqs.(8) and (9). The only difference between 2D electron systems and 3D ones is that FCQPT occurs at densities which are well below those corresponding to 2D systems. While in the bulk ^{3}He, FCQPT cannot probably take place being absorbed by the first order solidification.

Deriving Eq.(9), we assumed that the temperature $T = 0$. It is seen from Eq.(3) that the effective mass decreases when the temperature increases. The same is true when the system lays above the critical point.

Therefore, when T exceeds some temperature $T^*(x)$, Eq.(9) is no longer valid, and M^* depends on the temperature as well. To estimate $T^*(x)$, we can compare the deviation $\Delta x = |x - x_{FC}|$ with the deviation $\Delta x(T)$, generated by T. The deviation Δx can be expressed in terms of $M^*(x)$ using Eq.(9), $\Delta x/x \sim M/M^*(x)$. On the other hand, the temperature smoothing out the Fermi function $\theta(p_F - p)$ at p_F induces the variation $p_F \Delta p/M^*(x) \sim T$. As a result, we have $\Delta x(T)/x \sim \sim M^*(x)T/p_F^2$. Comparing these deviations, we find that at $T \geq T^*(x)$ the effective mass depends noticeably on the temperature, and the equation for $T^*(x)$ becomes

$$T^*(x) \sim p_F^2 \frac{M}{(M^*(x))^2} \sim \varepsilon_F(x) \left(\frac{M}{M^*(x)}\right)^2. \qquad (11)$$

Here $\varepsilon_F(x)$ is the Fermi energy of noninteracting electrons with mass M. From Eq. (11) it follows that M^* is always finite provided $T > 0$. We can consider $T^*(x)$ as the energy scale $e_0(x) \simeq T^*(x)$. This scale defines the area $(\mu - e_0(x))$ in the single particle spectrum where M^* is approximately constant, being given by $M^* = d\varepsilon(p)/dp$ [10]. According to Eqs.(9) and (11) it is easily verified that $e_0(x)$ can be written in the form

$$e_0(x) \sim \varepsilon_F \left(\frac{x - x_{FC}}{x_{FC}}\right)^2. \qquad (12)$$

At $T \ll e_0(x)$ and above the critical point the effective mass $M^*(x)$ is finite, the energy scale E_0 given by Eq.(4) vanishes and the system exhibits the LFL behavior. At temperatures $T \geq e_0(x)$ the effective mass M^* starts to depend on the temperature and the NFL behavior is observed. Thus, at $|x - x_{FC}|/x_{FC} \ll 1$ the system can be considered as a highly correlated one: at $T \ll e_0(x)$, the system is LFL, while at temperatures $T \geq e_0(x)$, the system possesses the NFL behavior. Then, it is clear that at $T \to 0$ the WF law is preserved. At $|x - x_{FC}|/x_{FC} \ll 1$, the effective mass given by Eq.(9) is very large, the Kadowaki-Woods ratio A/γ_0^2 is obeyed and the resistivity exhibits the T^2 behavior as it was demonstrated within a simple model of highly correlated liquid [17]. On the other hand, at $T \geq e_0(x)$, strong deviations from the T^2 behavior occur. We suppose that the resistivity follows a T^α dependence with $1 < \alpha < 2$ at $T \geq e_0(x)$. Here $\alpha = 1$ corresponds to strongly correlated liquid with FC, and $\alpha = 2$ corresponds to LFL [7, 17]. We remark that the outlined behavior was observed in several heavy fermion metals [15]. When the system's density x is outside the region $|x - x_{FC}|/x_{FC} \ll 1$, the scale e_0 becomes comparable with the Fermi level, the effective mass becomes $M^* \sim M$ and is approximately constant at energies, $(\mu - \varepsilon) \leq e_0$. Therefore the system becomes a normal Landau Fermi liquid.

We can expect to observe such a highly correlated electron (or hole) liquid in heavily overdoped high-T_c compounds which are located beyond the superconducting dome. Let us recall that beyond the FCQPT point the superconducting gap Δ_1 can be very small or even absent [18]. Indeed, recent experimental data have shown that this liquid does exist in heavily overdoped non-superconducting $La_{1.7}Sr_{0.3}CuO_4$ [6]. Note, that up to $T = 55$ K the resistivity exhibits the T^2 behavior, while at $T \geq 100$ K the resistivity follows a $T^{1.6}$ dependence [6]. Thus, we can estimate that $e_0(x) \sim 50$ K.

Now consider $M^*(B)$ as a function of a weak external magnetic field B at finite temperatures. The density x belongs to the area $(x - x_{FC})/x \ll 1$, and the electron system in question is the highly correlated one. The case when the system has undergone FCQPT was studied in [19]. This consideration will be applicable to any 2D or 3D electronic Fermi systems. The application of magnetic field B leads to a weakly polarized state, or Zeeman splitting, when some levels at the Fermi level are occupied by spin-up polarized quasiparticles. The width $\delta p = p_{F1} - p_{F2}$ of the area in the momentum space occupied by these quasiparticles is of the order

$$p_F \delta p/M^* \sim B\mu_{\text{eff}}. \qquad (13)$$

Here $\mu_{\text{eff}} \sim \mu_B$ is the electron magnetic effective moment, p_{F1} is the Fermi momentum of the spin-up electrons, and p_{F2} is the Fermi momentum of the spin-down electrons. As a result, the Zeeman splitting leads to the change Δx in the density x

$$\Delta x/x_{FC} \sim \delta p^2/p_F^2. \qquad (14)$$

We assume that $\Delta x/x_{FC} \ll 1$. Now it follows from Eqs.(9) and (14) that

$$M^*(B) \sim M\left(\varepsilon_F/B\mu_{eff}\right)^{2/3}. \qquad (15)$$

We note that M^* is determined by Eq.(15) as long as $M^*(B) \leq M^*(x)$, otherwise we have to use Eq.(9). It follows from Eq.(15) that the application of magnetic field reduces the effective mass. At finite temperatures $T \leq T^*(x)$, the effective mass is given by Eq.(15). At temperatures $T \sim T^*(x)$, both the magnetic field and temperature contribute to the decreasing of M^*. At $T^*(x) \ll T$, the effective mass is a diminishing function of the temperature. It is clear from Eq.(15) that $M^*(B)$ remains finite even at $x \to x_{FC}$ and $T \to 0$. In that case the effective mass $M^*(x)$ in Eq. (11) has to be substituted by $M^*(B)$. The behavior described above can be visualized by measuring the magnetic susceptibility

$\chi(T)$ at finite magnetic field so that $M^*(x) \geq M^*(B)$. The function $\chi(T)$ is a decreasing function of T at $T \sim e_0$. At $T \ll e_0$, the function $\chi(T)$ becomes independent of T and starts to depend on B, being a decreasing function of B.

Let us comment briefly on the problem of realization of the highly correlated liquid in dilute Fermi gases and in a low density neutron matter. We consider an infinitely extended system composed of Fermi particles, or atoms, interacting by an artificially constructed potential with the desirable scattering length a. These objects my be viewed as trapped Fermi gases, which are systems composed of Fermi atoms interacting by a potential with almost any desirable scattering length, similarly to that done for the trapped Bose gases, see e.g. [20]. If a is negative the system becomes unstable at densities $x \sim |a|^{-3}$, provided the scattering length is the dominant parameter of the problem. It means that $|a|$ is much bigger than the radius of the interaction or any other relevant parameter of the system. The compressibility $K(x)$ vanishes at the density $x_{c1} \sim |a|^{-3}$, making the system be completely unstable [21]. Expressing the linear response function in terms of the compressibility [22],

$$\chi(q \to 0, i\omega \to 0) = -\left(d^2E/d\rho^2\right)^{-1}, \qquad (16)$$

we obtain that the linear response function has a pole at the origin, $q \simeq 0$, $\omega \simeq 0$, at the same point x_{c1}. To find the behavior of the effective mass M^* as a function of the density, we substitute Eq.(7) into Eq. (10) taking into account that x_{cdw} is substiduted by x_{c1}, and $q_c/p_F \ll 1$ due to Eq.(16). At low momenta $q/p_F \sim 1$, the potential $v(q)$ is attractive because the scattering length is the dominant parameter and negative. Therefore, the integral on the right hand side of Eq.(10) is negative and diverges at $x \to x_{1c}$. The above consideration can be applied to clarification of the fact that effective mass M^* is again given by Eq.(9) with $x_{FC} < x_{c1}$. Note that the superfluid correlations cannot stop the system from squeezing, since their contribution to the ground state energy is negative. After all, the superfluid correlations can be considered as additional degrees of freedom which can therefore only decrease the energy. We conclude that the highly correlated behavior can be observed in traps by measuring the density of states at the Fermi level which becomes extremely large as $x \to x_{FC}$. At these densities the system remains stable because of $x_{FC} < x_{c1}$. It seems quite probable that the neutron-neutron scattering length ($a \simeq -20\,\mathrm{fm}$) is sufficiently large to make it the dominant parameter and to permit the neutron matter to have an equilibrium energy, equilibrium density, and the singular point x_{c1} at which the compressibility vanishes [23]. Therefore, we can expect FCQPT takes place in a low density neutron matter leading to the stabilization of the matter by lowering its ground state energy. A more detailed analysis of this issue will be published elsewhere.

To conclude, we have shown that our simple model based on FCQPT can explain the main features of the highly correlated liquid observed in different Fermi liquids. Thus, FCQPT can be viewed as an universal cause of the highly correlated behavior.

I am grateful to CTSPS for the hospitality during my stay in Atlanta. I also thank G. Japaridze for fruitful discussions. This work was supported in part by the Russian Foundation for Basic Research, # 01-02-17189.

1. K.-D. Morhard et al., Phys. Rev. **B53**, 2658 (1996).
2. A. Casey et al., J. Low Temp. Phys. **113**, 293 (1998).
3. A. A. Shashkin et al., Phys. Rev. **B66**, 073303 (2002).
4. M. Pfitzner and P. Wölfe, Phys. Rev. **B33**, 2003 (1986).
5. K. Kadowaki and S. B. Woods, Solid State Commun. **58**, 507 (1986).
6. S. Nakamae et al., cond-mat/0212283.
7. M. Ya. Amusia and V. R. Shaginyan, JETP Lett. **73**, 232 (2001); S. A. Artamonov and V. R. Shaginyan, JETP **92**, 287 (2001); M. Ya. Amusia and V. R. Shaginyan, Phys. Rev. **B63**, 224507 (2001); V. R. Shaginyan, Physica **B312-313C**, 413 (2002).
8. V. A. Khodel and V. R. Shaginyan, JETP Lett. **51**, 553 (1990); V. A. Khodel, V. R. Shaginyan, and V. V. Khodel, Phys. Rep. **249**, 1 (1994).
9. G. E. Volovik, JETP Lett. **53**, 222 (1991).
10. L. D. Landau, Sov. Phys. JETP **3**, 920 (1956).
11. M. Ya. Amusia, S. A. Artamonov, and V. R. Shaginyan, JETP Lett. **74**, 435 (2001).
12. R. B. Laughlin and D. Pines, Proc. Natl. Acad. Sci. U.S.A. **97**, 28 (2000).
13. P. W. Anderson, cond-mat/007185; cond-mat/0007287.
14. V. A. Khodel, V. R. Shaginyan, and M. V. Zverev, JETP Lett. **65**, 253 (1997).
15. C. M. Varma, Z. Nussinov, and Wim van Saarloos, Phys. Rep. **361**, 267 (2002).
16. V. R. Shaginyan, cond-mat/0208568.
17. V. A. Khodel and P. Schuck, Z. Phys. **B104**, 505 (1997).
18. M. Ya. Amusia and V. R. Shaginyan, Pis'ma Zh. Éksp. Teor. Fiz. **76**, 774 (2002).
19. Yu. G. Pogorelov and V. R. Shaginyan, Pis'ma Zh. Éksp. Teor. Fiz. **76**, 614 (2002).
20. S. Inouye et al., Nature **392**, 151 (1998).
21. M. Ya. Amusia, A. Z. Msezane, and V. R. Shaginyan, Phys. Lett. **A293**, 205 (2002).
22. L. D. Landau and E. M. Lifshitz, *Statistical Physics* I, Adison-Wesley, Reading, MA, 1970.
23. M. Ya. Amusia and V. R. Shaginyan, Eur. Phys. J. **A8**, 77 (2000).