Formation of solitons in pulsed NMR experiments in the A phase of He^3

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Spatially inhomogeneous solutions are found for the equations for spin dynamics of the A phase of He^3, which describe the steady-state magnetization precession and which are converted to planar solitons due to relaxation.

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The order parameter in the A phase of He^3 is characterized by two vectors: the spin vector **d** and the orbital vector **l**; their relative orientation in the bulk of the liquid is determined by the spin-orbit interaction with the energy $U = -\Omega_A^2/2\omega_L(\mathbf{l}, \mathbf{d})^2$. Here Ω_A is the frequency of longitudinal oscillations and ω_L is the Larmor frequency. U has two energy equivalent minima, $\mathbf{d} \parallel \mathbf{l}$ and $-\mathbf{d} \parallel \mathbf{l}$, because of which two types of domains can exist. The domain boundary with different relative orientation of **d** and **l** is the planar soliton in question. Since additional energy must be expended to form the domain wall, the single-domain state is advantageous, but the polydomain states must be specially prepared. It was determined experimentally that the domain walls in the A phase of He^3 are formed as a result of relaxation of magnetization to the equilibrium state after its large-angle deviation.[1–3] The goal of this paper is to explain theoretically this method of preparation of the soliton state.

Our explanation is based on the fact that the spatially homogeneous precession of magnetization in the A phase, as shown previously,[4] is unstable. If we drop the requirement of spatial homogeneity, then the steady-state precession of magnetization in a magnetic field, which satisfies the condition $\omega_L \gg \Omega_A$ in accordance with the results of Ref. 5 and in notations used therein, can be described by the following set of equations:

$$\frac{\partial V}{\partial \Phi} - \frac{\partial}{\partial x_i}\left(\frac{\partial G}{\partial \Phi_{,i}}\right) = 0, \tag{1}$$

$$S - 1 + \frac{\Omega_A^2}{8\omega_L^2}\,\zeta\,(\cos\beta - 1) = 0, \tag{2}$$

$$\frac{\partial}{\partial x_i}\left(\frac{\partial G}{\partial a_{,i}}\right) = 0, \tag{3}$$

$$\frac{1}{S\sin\beta}\left[\frac{\partial (V + G)}{\partial \beta} - \frac{\partial}{\partial x_i}\left(\frac{\partial G}{\partial \beta_{,i}}\right)\right] = \frac{\Omega_A^2}{8\omega_L^2}\,\zeta\,. \tag{4}$$

In the A phase,

$$V = -\frac{\Omega_A^2}{8\omega_L^2}\left[\cos^2\beta + \frac{1}{2}(1+\cos\beta)^2\cos 2\Phi\right] \quad (5)$$

and

$$G = \frac{c^2}{2\omega_L^2}\left[\frac{(1-\cos\beta)(3-\cos\beta)}{2}\alpha_{,i}\alpha_{,i} + \frac{\beta_{,i}\beta_{,i}}{2}\right.$$

$$\left. - 2(1-\cos\beta)\alpha_{,i}\Phi_{,i} + \Phi_{,i}\Phi_{,i}\right] \quad (6)$$

are the spin-orbit energy and the energy of inhomogeneity of the condensate, respectively, which are averaged over the fast motion and $\Omega^2\zeta/8\omega_L^2$ is the dimensionless shift of the precession frequency from the Larmor frequency. The solutions depending on one spatial coordinate x are of interest because of the planar solitons. Equations (1)-(4), for which $\alpha_{,x} = \Phi_{,x} = 0$, have a solution; Φ satisfies the equation $\partial V/\partial\Phi = 0$. The dependence of angle β on x is described by Eq. (4), which gives

$$\frac{(\beta')^2}{2} + \cos^2\beta + \frac{(1+\cos\beta)^2}{2} - \zeta\cos\beta = \frac{(\beta')^2}{2} + W(\cos\beta) = E = \text{const} \quad (7)$$

after integration and multiplication by $\beta'\sin\beta$. The prime denotes differentiation with respect to the dimensionless coordinate $\xi = \Omega x/c$, where c is the velocity of spin waves. Equation (7) is an energy integral for particle motion in the potential $W(\cos\beta)$. The solutions of this equation can be expressed in terms of elliptic integrals that depend on two constants ζ and E. The ζ constant determines the W potential and E determines the motion in the given potential. It is clear that all the solutions describe the periodic structures. To test the stability of the solutions, it is convenient to select other constants $-k = 2\pi/\lambda$, instead of ζ and E, where λ is the spatial period of the structure and $\mathcal{P} = 1/\lambda \int (S_z - S)d\xi$, where the integral is taken over the period of the structure. A direct calculation shows that the energy-density differential of the periodic structure, which is described by the solution of Eq. (7), has the following form in these variables

$$dF = -\omega_\perp d\mathcal{P} + \frac{\Omega_A^2}{8\omega_L^2} J\, dk, \quad (8)$$

where $J = 1/2\pi\oint\beta' d\beta$ is the mechanical action. According to the standard thermodynamic arguments,[6] the angular precession frequency $\omega_\perp$ and the action J in equilibrium must be constant in the entire investigated volume of He^3. The requirement that the second differential F should be positive gives rise to the following stability conditions of the found solutions:

$$\left(\frac{\partial}{\partial\zeta}\right)_k < 0, \quad \left(\frac{\partial J}{\partial k}\right)_\zeta > 0. \quad (9)$$

These conditions turn out to be satisfied only for the energies E which are larger than the largest maximum W. Such solutions correspond to a monotonic variation of the angle β with the coordinate. By definition of the spherical coordinates, the variation of β must be limited by the interval $(0, \pi)$; therefore, each time β crosses the value $m\pi$ (m is an integer), we can assume that the angle α varies discontinuously by a π. To preserve the continuity, the third Euler angle γ must vary by $(-1)^{m+1}\pi$. As a result, the angle $\Phi = \alpha + \gamma$ remains the same for even m and varies by 2π for odd m. As a result of relaxation, Φ is converted to an angle of rotation **d** with respect to **l**. A variation of Φ by 2π enables the domain walls to be formed. The domain walls, which correspond to the rotation of **d** by 2π, are unstable; they can either vanish or break up into two walls. What determines the selection of the option requires a more thorough analysis which takes the dissipation into account. If, however, the explanation of the formation process of solitons discussed here is accepted, then the variation of the NMR frequency shift observed experimentally[3] for large times can be attributed to the breakup of domain walls.

A transition from an unstable, homogeneous precession to a stable soliton precession apparently occurs in the form of a propagation of the front which is formed because of the initial difference in the precession frequencies at different points of the investigated He^3 volume. The planes on which $\beta = m\pi$ are the slip planes of the angle α, which makes it possible to determine the velocity of the front from a known value and from the uniform precession frequency. In fact, the phase difference in the precession, between the points 1 and 2, which builds up during the time t, between the points 1 and 2, which are located on the opposite sides of the front $\Delta\alpha = (\zeta_1 - \zeta_2)t\Omega_A^2/8\omega_L$, must form $n = \Delta\alpha/\pi$ slip planes of the phase, from which we obtain

$$v_{\mathrm{fr}} = \frac{\Omega_A}{8\pi\omega_L} \lambda (\zeta_1 - \zeta_2) c \quad , \tag{10}$$

for the propagation velocity of the front. The period λ of the structure is determined by the quantity $\mathscr{P}$. The order of magnitude of v_{fr} and its dependence on the temperature, pressure and magnetic field are determined by the combination of $\Omega c/\omega_L$. The transition from uniform precession to soliton precession can account for the fact that two frequencies of magnetization precession, which is deflected by a large angle, have been observed simultaneously in the experiment.[3] It should be borne in mind in a quantitative comparison with the experiment that the effect of dissipation on the magnetization motion was not taken into account in our analysis; moreover, we have analyzed only the initial stage of formation of solitons, which did not allow us to determine which soliton of all the possible types of solitons[7] is formed as a result of relaxation.

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