Baroclinic modification of the barotropic model of the large red spot of Jupiter

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Allowance for the vertical wave motion, which is attributable to the altitude gradient of the atmosphere density, makes it possible to bring the solution model of the red spot of Jupiter[1] into excellent quantitative agreement with the observational data.

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Petviashvili[1] proposed a model, according to which the large red spot of Jupiter is an anticyclonic Rossby soliton that drifts westward in the planet's atmosphere and rotates about its own axis in the opposite direction to the global rotation of the plan-

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et. His model differs from another soliton model of the spot[2] in its simplicity and accessibility to experimental verification.[3] This model is in qualitative agreement with the observations, but differs significantly from them quantitatively—with regard to the dimensions of the spot, its drift velocity (along a parallel), and the natural rotation period of the spot. Petviashvili's model is barotropic: it ignores the vertical wave motion, which is attributed to the vertical gradient of the atmospheric density. The purpose of this note is to show that a simple allowance for the vertical wave motion (i.e., the so-called baroclinic effect) in the soliton makes it possible to bring this model into good agreement with observational data.

The dispersion equation for Rossby waves in a shallow atmospheric layer of constant equivalent depth has the form[4]

$$\omega = \frac{\beta k_x}{k_x^2 + k_y^2 + k_z^2 f_o^2/N^2}, \tag{1}$$

where ω is the angular frequency of the wave, k_x, k_y, and k_z are the wave numbers of the oscillations along the latitude, the longitude, and the vertical, respectively, $f_0 = 2\Omega_0 \sin\phi$ is the Coriolis parameter, Ω_0 is the angular frequency of the planet's rotation, ϕ is the latitude, $\beta = -(2\Omega_0/R)\cos\phi$, R is the planet radius, and N is the Brunt-Väisälä frequency of the vertical oscillations of the medium, which is stable with respect to convection

$$N^2 = -\frac{g}{\rho}\frac{d\rho}{dz} - \frac{g^2}{c_s^2}, \tag{2}$$

g is the free-fall acceleration and c_s is the velocity of sound. For an isothermal atmosphere, whose density $\rho(z)$ decreases vertically in accordance with the Boltzmann law with equivalent height H_0, we obtain the following upper estimate of N:

$$N = \left[\frac{(\gamma - 1)}{\gamma}\frac{g}{H_o}\right]^{1/2}, \tag{2'}$$

where T is the temperature of the atmosphere and $\gamma = 1.4$ is the adiabatic exponent. The following varieties of Rossby waves satisfy Eq. (1)[4]: the *barotropic* mode, which exists irrespective of $d\rho/dz$ and which is the only one taken into account in the model of Ref. 1:

$$k_z^{(o)} = \left(\frac{N^2}{gH_o}\right)^{1/2}, \tag{3}$$

and the *baroclinic* modes (which are ignored in the model of Ref. 1):

$$k_z^{(m)} = \frac{m\pi}{H_o}, \quad m = 1, 2, 3, \ldots \tag{4}$$

The dispersion equation (1) has the same form for all modes

$$\omega = \frac{\beta k_x}{k_x^2 + k_y^2 + 1/r_R^2} \quad ; \tag{5}$$

for the barotropic mode $r_R = r_e$ and for the baroclinic modes $r_R = r_i$, where the parameters r_e and r_i are, respectively, the external and internal Rossby radii

$$r_e = \frac{(gH_o)^{1/2}}{f_o} \quad , \quad r_i = \frac{NH_o}{m\pi f_o} \quad . \tag{6}$$

In the barotropic model[1] the soliton diameter $2a$, its drift velocity V_x relative to the planet, and the natural rotation period of the soliton $T_r = 2\pi/\omega_r$ are defined by the relations

$$\left.\begin{aligned} 2a &\approx 3{,}5\, r_e\, (h)^{-1/2}, \\ V_x &\gtrsim \beta r_e^2 \quad , \\ \frac{\omega_r}{f_o} &\approx \frac{1}{3} h^2 \;{}^{1)}, \end{aligned}\right\} \tag{7}$$

where $h = \Delta H/H_0$ is the relative elevation (amplitude) of the soliton.

In the baroclinic modification of this model we must replace r_e by r_i, i.e., we must assume that

$$\left.\begin{aligned} 2a &\approx 3{,}5\, r_i\, (h)^{-1/2}, \\ V_x &\gtrsim \beta r_i^2 \quad , \\ \frac{\omega_r}{f_o} &\approx \frac{1}{3} h^2 \left(\frac{r_e}{r_i}\right)^2 . \end{aligned}\right\} \tag{8}$$

The differences in the parameters of the barotropic and baroclinic solitons are determined by the ratio r_e/r_i (to different degrees). According to Eqs. (2′) and (6), this ratio is (for $m = 1$)

$$\frac{r_e}{r_i} \gtrsim \left(\frac{\pi^2 \gamma}{\gamma - 1}\right)^{1/2} \approx 6. \tag{9}$$

The following values were obtained for Jupiter and the large red spot from observation: $\Omega_0 = 1.74 \times 10^{-4}$ sec^{-1}, $\omega_r \approx 1.1 \times 10^{-5}$ sec^{-1}, $R = 71\,000$ km, $g = 2.5 \times 10^3$ cm · sec^{-2}, latitude of spot $\phi = -22°$, $T = 130$ K, $H_0 \approx 20$ km, the spot diameter is 10 000 km along a meridian and 20 000 km along a parallel, $N = 2 \times 10^{-2}$ sec^{-1} [see also Eq. (2′)], the westward drift velocity of the spot is about 3 m/sec, and the amplitude of the anticyclone in the spot (the average of two measurements in Ref. 6) is $h = 10^{-1}$. From these numbers and from Eq. (6) we have: $r_e \approx 6\,000$ km and $r_i \approx 1\,000$ km ($m = 1$). These data were used to compile Table I.

TABLE I.

	r_e/r_R	$2a$, 10^3km	V_x, m/sec	ω_r/f_0
Barotropic model[1]	1	66	160	3×10^{-3}
Baroclinic model, $m = 1$	6	11	4,5	11×10^{-2}
Observations	—	10 – 20	3	9×10^{-2}

It can be seen from Table I that the observed spot size is 3–7 times smaller, the drift velocity is 50 times smaller, and the natural rotation frequency is 30 times greater than that predicted by the model.[1] At the same time, the baroclinic modification of this model for $m = 1$ agrees well with the results of the observation. It would be interesting to compare the parameters of the smaller Jovian spots with the higher-order baroclinic modes ($m > 1$).

[1)] From Eq. (5) of Ref. 1 we have: $\omega_r a \approx f_0 r_e^2 h/a$, from which $\omega_r/f_0 \approx r_e^2 h/a^2 \approx (r_e^2/r_R^2)(h^2/3)$, where $r_R = r_e$ for the barotopic mode and $r_R = r_i$ for the baroclinic modes.

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