Collective excitations in a Bose gas of a spin-polarized atomic hydrogen

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A two-component Bose condensate and collective excitations in a spin-polarized Bose gas of an atomic hydrogen are analyzed. The ratios for critical gas density are calculated. The spin excitations are threshold-free if this critical density is exceeded.

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1. Atomic hydrogen in a magnetic field is a four-component Bose gas, whose individual components have different spin configurations. Since this system remains a gas even at $T=0$, the condensate and the collective excitations in it can be adequately analyzed by using the well-known Bogolyubov method.[1] The excitations, in which the electron spin is reversed, are of particular interest, since the interaction of such particles with the polarized-condensate atoms has a singlet component, which reduces the excitation energy compared with the gap $2\mu_B H$. This imposes constraints on the gas density—theoretically, there is a critical density n_c, at which the energy gap, which impedes depolarization and hence recombination of atomic hydrogen, vanishes.

The excitations in such a gas were analyzed for the first time by Berlinsky[2] (see also Ref. 3), who assumed that the condensate has one component. However, the results obtained by Berlinsky are incorrect because of the implicit assumption that the pair correlation functions for the atoms with parallel electron spins are the same as those for the atoms with antiparallel electron spins. In fact, there is a basic difference between these functions, and, as we shall see from the results below, the criterion for critical density turns out to be entirely different. Allowance for the two-component nature of Bose condensate changes the spin excitations.

2. The system of spin wave functions of an isolated hydrogen atom in a strong magnetic field has the form

$$\phi_1 = \alpha\left(\frac{1}{2}\right)\beta\left(\frac{1}{2}\right)$$

$$\phi_2 = \alpha\left(\frac{1}{2}\right)\beta\left(-\frac{1}{2}\right) + \kappa\alpha\left(-\frac{1}{2}\right)\beta\left(\frac{1}{2}\right) \qquad (1)$$

$$\phi_3 = \alpha\left(-\frac{1}{2}\right)\beta\left(-\frac{1}{2}\right)$$

$$\phi_4 = \alpha\left(-\frac{1}{2}\right)\beta\left(\frac{1}{2}\right) - \kappa\alpha\left(\frac{1}{2}\right)\beta\left(-\frac{1}{2}\right),$$

where $\alpha(\sigma')$ and $\beta(\sigma'')$ are the spin wave functions of the electron and nucleus, respectively, σ' and σ'' are the projections of the electron and nuclear spins in the direction of the magnetic field, $\kappa = A/4\mu_B H \ll 1$, and A is the hyperfine interaction constant.

At $T=0$ most of the particles in a polarized gas of limited density n ($na_t^3 \ll 1$, where a_t is the scattering length due to collision of atoms in the triplet state) are in the Bose condensate that consists of a mixture of atoms in the ϕ_3 and ϕ_4 states. Although the energy in the ϕ_4 state is slightly lower than that in the ϕ_3 state (at $H \approx 10^5$ Oe $\Delta\epsilon \approx 5 \times 10^{-2}$ K), the transition time from the ϕ_3 state to the ϕ_4 state due to dipole-dipole interaction in the collision turns out to be large compared with the typical decay time of a polarized system (see Ref. 4 for a more detailed account). We can assume, therefore, that both states can have a two-component Bose condensate with a fixed number of particles N_3 and N_4.

The subspace of the ϕ_1 and ϕ_2 states is separated from the Bose condensate by an energy gap whose scale is $2\mu_B H$. When a particle (or a few particles) is produced in the ϕ_1 and ϕ_2 states, the number of particles in the upper (ϕ_1, ϕ_2) and lower (ϕ_3, ϕ_4) subspace is conserved in the exchange interaction with the condensate.

The Hamiltonian of the pair interaction of particles has the usual structure

$$\hat{H}_{int} = U_l(R) + \Delta(R)\left(\hat{S}_1\hat{S}_2 + \frac{1}{4}\right), \qquad (2)$$

where $\mathbf{S}_i$ are the operators of the electron spins of particles.

We shall begin by analyzing the collision of a ϕ_1 particle with the condensate atoms, ignoring the small corrections of order κ. As a result of collision of particles with opposite electron spins, the wave function of the pair is a superposition of singlet and triplet functions. If a ϕ_1 particle is scattered by a ϕ_4 particle, the scattering can occur only in the triplet channel as $T \to 0$, since the nuclear spins are parallel to each other (the scattering in the singlet channel occurs only for odd values of the orbital quantum number j, and the corresponding amplitude vanishes when the particle energy approaches zero). Thus the scattering is purely elastic, and the particles remain in the ϕ_1 and ϕ_4 states. The corresponding vertex for the particle-interaction Hamiltonian in the second quantization is

$$\frac{2\pi\hbar^2}{mV} a_t . \qquad (3)$$

The interaction of ϕ_1 and ϕ_3 particles can have two elastic-scattering channels. This is attributable to the fact that the nuclear spins of particles in this case have opposite projections, which gives rise to the possibility of s scattering in the triplet and singlet states. The corresponding vertex is

$$\frac{2\pi\hbar^2}{mV} \frac{(a_t + a_s)}{2}, \qquad (4)$$

where a_s is the scattering length in the singlet state.

However, the amplitude of a different process, in which the ϕ_3 condensate particle changes to the ϕ_4 state and the ϕ_1 particle changes to the ϕ_2 state, turns out to be nonvanishing after the collision of ϕ_1 and ϕ_3 particles. The vertex for this process is determined by the expression

$$\frac{2\pi\hbar^2}{mV}\,\frac{(a_t - a_s)}{2}\,. \tag{5}$$

We have a mirror picture when a ϕ_2 particle interacts with a condensate—the scattering of ϕ_2 and ϕ_3 particles is purely elastic with the vertex (3), and the interaction of ϕ_2 and ϕ_4 particles leads to elastic scattering with the vertex (4), as well as to the transition of the particles to the ϕ_1 and ϕ_3 states, which is described by the vertex (5).

We shall write the Hamiltonian in the second quantization for particles in the ϕ_1 and ϕ_2 states, which interact with the Bose-condensate background, and take advantage of the change to the c numbers for the operators of the creation and annihilation of particles in the condensate. Incorporating Eqs. (3)-(5), we directly determine

$$H = \sum_k \omega_1(k)\, a^+_{1k} a_{1k} + \sum_k \omega_2(k)\, a^+_{2k} a_{2k} + \sum_k \gamma_{12}[\, a^+_{2k} a_{1k} + a^+_{1k} a_{2k}\,]. \tag{6}$$

Here

$$\omega_1(k) = \frac{\hbar^2 k^2}{2m} + \tilde{\epsilon}_1 + \frac{4\pi\hbar^2}{m}\left[n_4 a_t + n_3 \frac{a_t + a_s}{2}\right],$$

$$\omega_2(k) = \frac{\hbar^2 k^2}{2m} + \tilde{\epsilon}_2 + \frac{4\pi\hbar^2}{m}\left[n_3 a_t + n_4 \frac{a_t + a_s}{2}\right],$$

$$\gamma_{12} = \frac{2\pi\hbar^2}{m}(a_t - a_s)\sqrt{n_3 n_4}\,; \tag{7}$$

$n_i = N_i/V$, and ϵ_i is the energy of a hydrogen atom in the magnetic field, which corresponds to the ϕ_i state in (1).

The Hamiltonian (6) can be reduced to the diagonal form by canonically transforming it to the new Bose operators

$$\begin{cases} b_{1k} = u_k a_{1k} - v_k a_{2k}, \\ b_{2k} = v_k a_{1k} + u_k a_{2k}, \end{cases} \qquad u_k^2 + v_k^2 = 1\,. \tag{8}$$

The Hamiltonian corresponding to the new, collective excitations has the form

$$H = \sum_k [\,\epsilon_-(k)\, b^+_{1k} b_{1k} + \epsilon_+(k)\, b^+_{2k} b_{2k}\,], \tag{9}$$

$$\epsilon_-(k) = \omega_1 u_k^2 + \omega_2 v_k^2 - 2\gamma_{12} u_k v_k; \qquad \epsilon_+(k) = \omega_1 v_k^2 + \omega_2 u_k^2 + 2\gamma_{12} u_k v_k,$$

where

$$u_k^2 = \frac{1}{2} + \frac{1}{2}\sqrt{\frac{\xi^2}{4+\xi^2}}\,; \qquad \xi^2 = \left(\frac{\omega_1 - \omega_2}{\gamma_{12}}\right)^2. \tag{10}$$

3. Let us examine the case $n_3 = n_4$. In this case $\xi = |\tilde{\epsilon}_1 - \tilde{\epsilon}_2|/\gamma_{12} \approx |A/2 - 2\mu_p H|/\gamma_{12}$ ($\mu_p > 0$ is the magnetic moment of a proton). If the particle density is sufficiently high, so that $\xi \ll 1$, we can determine from Eqs. (9) and (10),

$$\epsilon_-(k) = \frac{\hbar^2 k^2}{2m} + \frac{\tilde{\epsilon}_1 + \tilde{\epsilon}_2}{2} + \frac{4\pi\hbar^2}{m} n a_t, \tag{11}$$

$$\epsilon_+(k) = \frac{\hbar^2 k^2}{2m} + \frac{\tilde{\epsilon}_1 + \tilde{\epsilon}_2}{2} + \frac{2\pi\hbar^2}{m}(a_t - a_s).$$

The ensuing collective excitations are a superposition of the ϕ_1 and ϕ_2 states. It is easy to see that this leads to the appearance in these excitations of a constant component of the magnetic moment, which is perpendicular to the magnetic field and which is

$$M_x = \pm(\mu_p + \kappa\mu_B).$$

(We note that it was pointed out in Ref. 5 that a transverse macroscopic moment can occur in a thermodynamically unstable, two-component Bose condensate.)

The excitation energy corresponding to the transition of a particle from the condensate to the $1(\epsilon_1)$ state, or to the $2(\epsilon_2)$ state is given by

$$\epsilon_1(k) \cong \epsilon_-(k) - \frac{4\pi\hbar^2}{m} n a_t + \mu_B H \cong \frac{\hbar^2 k^2}{2m} + 2\mu_B H, \tag{12}$$

$$\epsilon_2(k) \cong \epsilon_+(k) - \frac{4\pi\hbar^2}{m} n a_t + \mu_B H \cong \frac{\hbar^2 k^2}{2m} + 2\mu_B H - \frac{2\pi\hbar^2}{m} n(a_t - a_s).$$

It follows from expressions (13) that the inequality

$$\frac{2\pi\hbar^2}{m} n(a_t - a_s) < 2\mu_B H \tag{13}$$

must be satisfied in the absence of nonthreshold spin excitations.

For finite densities when the inverse inequality $\xi \gg 1$ is valid

$$\epsilon_1(k) \cong \epsilon_2(k) \cong \frac{\hbar^2 k^2}{2m} + 2\mu_B H - \frac{\pi\hbar^2}{m} n(a_t - a_s). \tag{14}$$

As a result, inequality (13) must hold for both types of excitation if the multiplier 2 on the left-hand side of this inequality is dropped.

If $n_3 \ll n_4$, then the excitation energies remain the same as in (12), and $\epsilon_1(k) \leftrightarrow \epsilon_2(k)$ inversion occurs at $n_3 \gg n_4^4$. Thus the criterion (13) remains valid.

The criteria determined by us, which turned out to be appreciably weaker in comparison with those of Berlinsky,[2] lead to much higher values of critical density (according to the calculations of Ref. 4, $a_t = 0.72$ Å and $a_s = 0.33$ Å). In a strong magnetic field these criteria are realized with a large reserve for all conceivable densities of the gas phase.

1. N. N. Bogolyubov, J. Phys. (USSR) 11, 23 (1947).
2. A. J. Berlinsky, Phys. Rev. Lett. 39, 359 (1977).
3. A. J. Berlinsky, R. D. Etters, V. V. Goldman, and I. F. Silvera, Phys. Rev. Lett. 39, 356 (1977).
4. Yu. Kagan, I. A. Vartan'yants, and G. V. Shlyapnikov, Zh. Eksp. Teor. Fiz. 81, 1113 (1981).
5. E. D. Siggia and A. E. Ruckenstein, Phys. Rev. Lett. 44, 1423 (1980).

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