Parametric mechanisms for detecting gravitational waves

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An intense electromagnetic wave and a gravitational wave can interact to effectively generate electromagnetic waves at sum and difference frequencies. The self-effect of a monochromatic electromagnetic wave through a gravitational field leads to third-harmonic generation.

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1. A vacuum with a static gravitational field is known to be equivalent to a medium with a refractive index which is determined by the strength of the gravitational field.[1] If, on the other hand, a gravitational wave is propagating in a vacuum, then there are also changes in the electrodynamic properties; they become periodic in the spatial and temporal variables.[1)] The problem of the propagation of an electromagnetic wave in a vacuum with a gravitational wave thus reduces to a parametric problem. A circumstance of fundamental importance here is that the propagation velocities of the electromagnetic and gravitational waves are equal,[1-4] so that there may be a resonant collinear interaction when the resonance conditions are satisfied exactly. Although the interaction itself may be slight, the waves which appear will have amplitudes that grow linearly over distance, and over a large distance the effect can become quite significant.

As an electromagnetic wave propagates in a vacuum in which a gravitational wave is also propagating in the same direction, new electromagnetic waves appear at the sum and difference frequencies, with amplitudes that grow over distance. The detection of waves at the sum and difference frequencies might be used as a method for detecting gravitational waves.

Let us derive an expression for the amplitudes of the electromagnetic waves at the sum and difference frequencies which are "diffracted" by a gravitational wave.

Assuming that the gravitational field h_{ik} is weak and that the metric tensor is $g_{ik} = g_{ik}^{(0)} + h_{ik}$, where $g_{ik}^{(0)}$ are the Galilean values ($g_{\alpha 0}^{(0)} = 0, g_{00}^{(0)} = 1, g_{\alpha\beta}^{(0)} = -\delta_{\alpha\beta}$), we can write Maxwell's equations incorporating gravitation as follows:

$$\begin{aligned} &\mathrm{rot}\,\mathbf{H} - \frac{1}{c\sqrt{\gamma}}\frac{\partial}{\partial t}(\sqrt{\gamma}\,\mathbf{D}) = 0, \quad \mathbf{D} = \mathbf{E}/\sqrt{g_{00}}, \\ &\mathrm{rot}\,\mathbf{E} + \frac{1}{c\sqrt{\gamma}}\frac{\partial}{\partial t}(\sqrt{\gamma}\,\mathbf{B}) = 0, \quad \mathbf{B} = \mathbf{H}/\sqrt{g_{00}}, \end{aligned} \tag{1}$$

where **E** and **H** are the electric and magnetic fields. The gravitational field enters Eqs. (1) only through the determinant of the spatial part of the metric tensor, $\gamma_{\alpha\beta} = -g_{\alpha\beta} = \delta_{\alpha\beta} - h_{\alpha\beta}$, i.e., $\gamma = \mathrm{Det}\,|\gamma_{\alpha\beta}|$. By virtue of the polarization properties of a gravitational field which is propagating along the x^1 direction, the only nonvanishing components $h_{\alpha\beta}$ are h_{23} and $h_{33} = -h_{22}$ (Sec. 102 of Ref. 1), and the determinant γ turns out to be $1 - h_{22}^2 - h_{23}^2$, i.e., quadratic in the strength of the gravitational field. For the same reason, the effect discussed below turns out to be quadratic in the $h_{\alpha\beta}$. Furthermore, it is easy to show that the incorporation in the tensor g_{ik} of terms proportional to the square amplitude of the gravitational wave, $h^0_{\alpha\beta}$, leads to the appearance in the determinant γ of terms that are proportional to $h^{03}_{\alpha\beta}$ and $h^{04}_{\alpha\beta}$, which are small. Assuming the gravitational wave to be given,

$$h_{\alpha\beta} = h^{\circ}_{\alpha\beta}\, e^{i\omega_g\left(t - \frac{x^1}{c}\right)} + \text{c.c.}, \tag{2}$$

where $\omega_g = k_g c$ is the frequency of the gravitational wave, and writing the electromagnetic field as

$$\mathbf{E} = \mathbf{E}_0 + \mathbf{E}', \quad \mathbf{E}_0 = \vec{\epsilon}_0\, e^{i\omega_0\left(t - \frac{x^1}{c}\right)} + \text{c.c.}; \quad \mathbf{E}' = \vec{\epsilon}\,'(x^1)\, e^{i\omega'\left(t - \frac{x^1}{c}\right)} + \text{c.c.}, \tag{3}$$

where $\vec{\epsilon}_0$ and $\vec{\mathscr{H}}_0$ are the amplitudes of the given intense electromagnetic wave, and $\vec{\epsilon}$ and $\vec{\mathscr{H}}'$ are those of the weak "diffracted" wave, we find from (1) an equation for the field E' with a given polarization on the right side:

$$\left(\text{rot rot} + \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right)\mathbf{E}' = -\frac{1}{2c^2}\frac{\partial}{\partial t}\left(\mathbf{E}_0\frac{\partial}{\partial t}\gamma\right) - \frac{1}{2c}\left[\mathbf{H}_0, \text{grad}\,\frac{\partial \gamma}{\partial t}\right]. \tag{4}$$

It can be seen from (4) that under the resonance conditions $\omega_0 \pm 2\omega_g = \omega'$ (for a collinear interaction the conservation law for the wave vectors is satisfied automatically, since the velocities of the electromagnetic and gravitational waves are equal[4]), there exists a solution which grows over distance along the wave propagation direction, x^1. Simple calculations (as in nonlinear optics[5]) of the amplitude of the electromagnetic wave which is generated under the boundary condition $\epsilon'(x^1 = 0) = 0$ yield

$$\epsilon_\pm \equiv \epsilon'(x^1 = L) = \frac{i(h^{\circ 2}_{22} + h^{\circ 2}_{23})\omega_g L}{2c}\;\frac{\omega_0}{\omega_0 \pm 2\omega_g}\,\epsilon_0 \qquad (\epsilon_0,\, h^{\circ}_{\alpha\beta} = \text{const}). \tag{5}$$

Here L is the distance which the intense electromagnetic wave has traversed along with the gravitational wave, and the polarizations of the diffracted waves, $\mathbf{E}'$, and of the initial wave, $\mathbf{E}_0$, are the same. Noting that the energy flux along the x^1 axis is $\mathscr{P} = c^3\omega_g^3(h^{02}_{22} + h^{02}_{23})/4\pi\kappa$ for a plane gravitational wave, where κ is the gravitational constant, we find from (5) the relative amplitudes of the electromagnetic waves at the sum and difference frequencies:

$$\left|\frac{\epsilon_\pm}{\epsilon_0}\right| = \frac{2\pi\kappa \mathscr{P} L}{c^4 \omega_g} \frac{\omega_0}{\omega_0 \pm 2\omega_g} . \tag{6}$$

Consequently, the detection of gravitational waves reduces to the observation of satellites on the signal from a coherent source of electromagnetic radiation if the intensity of these satellites increases linearly with distance from the source. For example, for gravitational waves with an energy flux density $\mathscr{P} \sim 10^{18}$ erg/(cm · s) at the frequency $\omega_g \sim 2\pi \cdot 100$ Hz coming from a source at a distance $L \sim 5 \times 10^{22}$ cm (in the Crab Nebula), the amplitude of the sum or difference peak in (6) is 10^{-9} of the amplitude of the light source, ϵ_0 ($\omega_0 \gg \omega_g$). Clearly, the effect is possible if there is no deviation from the resonance conditions anywhere along the interaction path. A deviation might be caused, for example, by a difference of the refractive index of the medium, $n(\omega)$, from unity or by a frequency dispersion of this index, i.e., if

$$\frac{2\omega_g L}{c}\left[1 - n(\omega_0) + \omega_0 \left.\frac{\partial n}{\partial \omega}\right|_{\omega_0}\right] << 1 \qquad (\omega_0 >> \omega_g) . \tag{7}$$

Assuming $n(\omega) = 1 - 2\pi e^2 N/M\omega_0^2$ (as for a plasma), where N is the number of particles per unit volume with charge e and mass M, we find that condition (7) is satisfied with $N \ll 10^9$ cm^{-3} for protons and with $N \ll 10^5$ cm^{-3} for electrons with $L = 5 \times 10^{22}$ cm and $\omega_g = 2\pi \cdot 100$ Hz at visible wavelengths.

2. There is yet another, extremely interesting, consequence of a parametric self-effect of an electromagnetic wave in a vacuum through stimulated waves. For an intense, monochromatic electromagnetic signal in a vacuum, this self-effect leads to harmonics which can be detected experimentally, as we will now show.

From the theory of relativity for a weak gravitational field,[1] we have

$$\Box h_{ik} = \frac{16\pi\kappa}{c^4}\left(T_{ik} - \frac{1}{2} g_{ik} T\right), \tag{8}$$

where T_{ik} is the energy–momentum tensor of the electromagnetic field. After a derivation similar to that above (Section 1), we find the following result for a nontransverse stimulated gravitational wave which is generated by an intense electromagnetic wave of frequency ω_0 that is propagating along the x^1 axis:

$$h_{ik} = (i4\pi\kappa W_0/c^3\omega_0)\left(e^{i\,2\omega_0\left(t - \frac{x^1}{c}\right)} + \text{c.c.}\right) \qquad (i, k = 0,1). \tag{9}$$

In Eqs. (8) and (9) we have used the components $T_{00} = T_{01} = T_{10} = T_{11} = W[T = T_i^i = T_{ik}(i, k \neq 0) = 0]$, $W = E_0^2/4\pi$, $W_0 = \epsilon_0^2/4\pi$ (Ref. 1). It can be seen from (9) that the amplitude of the gravitational wave increases linearly with increasing interaction length. Substituting (9) into Maxwell's equations in (1), and noting that we have $\gamma = 1 - h_{11}$ in this case, we write the following equation for the resonant electromagnetic wave at the third harmonic, $\epsilon_{3\omega_0}$:

$$\left| \frac{\epsilon_{3\omega_0}}{\epsilon_0} \right| = \frac{\pi \kappa W_0 L^2}{3c^4} , \qquad (10)$$

where L is the distance traversed by the intense electromagnetic wave from its source. The third-harmonic amplitude is independent of the frequency, being determined by only the energy density and the distance over which the self-effect operates. Equations (10) and (6) are the same in structure. The energy density of the gravitational wave is replaced in (10) by the energy density of the electromagnetic wave, and the distance is raised to a power one unit higher because of the "double" resonance: The nontransverse gravitational waves which are generated are at resonance with the initial electromagnetic wave, and the gravitational wave and the intense electromagnetic wave in turn resonantly generate an electromagnetic wave at the tripled frequency.

Estimates based on Eq. (10) show that with a detector on the moon ($L \cong 3.8 \times 10^{10}$ cm) capable of detecting several quanta per second in an area of 1 cm^2 the electric field in an electromagnetic pulse coming from the earth would have to be of the order of 10^7 V/cm.

The generation of the third harmonic of a monochromatic electromagnetic signal in a vacuum is essentially yet another consequence of the general theory of relativity.

1)Sibgatullin[2] and Dinariev and Sibgatullin[3] have analyzed the propagation of weak electromagnetic and gravitational waves in an intense electromagnetic field.

1. L. D. Landau and E. M. Lifshitz, Teoriya Polya, Nauka, Moscow, 1967 (Classical Theory of Fields, Addison-Wesley, Reading, Mass. 1971).
2. N. R. Sibgatullin, Zh. Eksp. Teor. Fiz. **66**, 1187 (1974) [Sov. Phys. JETP **39**, 579 (1974)].
3. O. Yu. Dinariev and N. R. Sibgatullin, Zh. Eksp. Teor. Fiz. **72**, 1231 (1977) [Sov. Phys. JETP **45**, 645 (1977)].
4. A. S. Kompaneets, Zh. Eksp. Teor. Fiz. **37**, 1722 (1959) [Sov. Phys. JETP **37**, 1218 (1960)].
5. N. Bloembergen, Nonlinear Optics, Benjamin, New York, 1965 (Russ. transl., Mir, Moscow, 1966).

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