Coulomb systems with a short-range interaction: the $\Sigma^- p$ atom and resonant levels of light nuclei

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An equation relating the shift and width of atomic levels to the strong scattering length, which was derived previously {T. L. Trueman, Nucl. Phys. **26**, 57 (1961); A. E. Kudryavtsev and V. S. Popov, Pis'ma Zh. Eksp. Teor. Fiz. **29**, 311 (1979) [JETP Lett. **29**, 280 (1979)]; V. S. Popov, A. E. Kudryavtsev, and V. D. Mir, Zh. Eksp. Teor. Fiz. **77**, 1727 (1979) [Sov. Phys. JETP **50**, 865 (1979)]}, is generalized to the case of complex momenta k and arbitrary values of the angular momentum l and of $\zeta = -Z_1Z_2$. The shift and width of the s levels of the $\Sigma^- p$ atom are calculated. The relationship between the position and width of the resonances in the case of Coulomb repulsion is studied.

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There is increasing interest in systems which are bound by a Coulomb potential with a short-range distortion. In the case of an attractive Coulomb potential, the spectrum of such a system may undergo a reconstruction[3,4] which puts the atomic levels in positions greatly different from those of the unperturbed Coulomb problem. There are indications that the situation in the pp atom corresponds approximately to the reconstruction described in Refs. 2 and 5. Numerical calculations in the atomic physics of rare-earth elements indicate a collapse of the wave function of the electrons of the $4f$ shell,[6] which is characteristic of a reconstruction of atomic levels with $l \neq 0$ (Ref. 7).

In the present letter we wish to call attention to certain other Coulomb systems for which the strong interaction may be important at distances $r \sim r_0 \ll a_B$. These systems should be described by the equation

$$\left\{ \lambda + 2\zeta\left[\psi(1-\zeta/\lambda) + \ln\frac{\lambda}{|\zeta|}\right]\right\} \prod_{j=1}^{l}\left(\frac{\zeta^2}{j^2} - \lambda^2\right) = \frac{1}{a_{cs}^{l}} + \frac{1}{2} r_{cs}^{l}\lambda^2, \tag{1}$$

which relates the level spectrum to the scattering length a_{cs}^l and the effective radius r_{cs}^l. Here $\hbar = m = e = 1$, $\lambda = ik$ is the imaginary momentum (the energy is $E = -\lambda^2/2$), $\zeta = Z_1 Z_2$, a_{cs}^l is the effective radius.[1)]

We will apply Eq. (1) to the $\Sigma^- p$ atom. The scattering length for $\Sigma^- p$ scattering with $l = 0$ is $a_s(\Sigma^- p \to \Sigma^- p) = 2/3a_{1/2} + 1/3a_{3/2}$, where a_I is the scattering length for ΣN scattering with isospin I. Since there are indications that $a_{1/2}$ is large, a_s may also turn out to be large. Information on $a_{1/2}$ can be extracted from data on the (Λp)

TABLE I.

$a_{1/2}(\Sigma N)$	Author	$a_{cs}(\Sigma^-p)$	ΔE, keV	$\frac{1}{2}\Gamma$, keV
$-2.6-1.9i$	A. M. Badalyan	$-1.8-1.6i$	-2.0	2.1
$-1.73-1.96i$	—,,—	$-1.1-1.5i$	-1.2	1.9
$7.5-3.0i$	T.Tan	$3.9-1.1i$	3.3	0.7
$3.3-1.6i$	R.H.Dalitz	$2.0-0.8i$	1.9	0.7
$-2.4-1.75i$	—,,—	$-1.7-1.4i$	-1.9	1.9
$-3.0-1.8i$	J J .de Swart	$-2.2-1.6i$	-2.5	2.1

mass spectrum near the ΣN threshold from the reaction $K^-d \to \pi^-\Lambda p$. In this reaction, a peak with a mass of 2129 MeV is observed in the mass spectrum of the Λp system ($I = 1/2$) (Ref. 8). This peak has been interpreted[9-11] as a cusp intensified by the large amplitude for the YN interaction ($Y = \Sigma, \Lambda; I = 1/2, S = 1$). Different theoretical calculations lead to very different results for the amplitude $a_{1/2}(\Sigma N)$, as can be seen from the first column in Table I. These differences lead in turn to different interpretations of the nearest singularity of the ΣN scattering amplitude. We will show below that a study of the shifts and widths of the s levels of the Σ^-p atom can yield useful information on the amplitude $a_{1/2}(\Sigma N)$.

Table I shows the energies $E_{1s} = E_0 + \Delta E - i\Gamma/2$ ($E_0 = -14.0$ keV) calculated for several characteristic scattering lengths $a_{1/2}$ which have been given in the literature. We used Eq. (1) with $l = 0$ and $\zeta = 1$, ignoring the amplitude $a_{3/2}$. We note that ΔE and Γ are essentially independent of the radius r_{cs}, which was varied from 0 to 3 F. For none of the $a_{1/2}$ fits does the shift of the 1s level turn out to be small, so this shift appears to be measurable. Such measurements would reveal the nature of the singularity in the ΣN amplitude.[2)] A few comments are in order here.

1. It is extremely important to use Eq. (1) in the calculation of ΔE_{ns} and Γ_{ns}. The perturbation-theory equation

$$\Delta E_{ns} - \frac{i}{2}\Gamma_{ns} = \frac{2}{n^3} a_s \qquad (2)$$

yields the width Γ_{1s} within an error amounting to a factor of two or three. The error in the determination of the level shift is smaller (~20%).

2. In observation of the shifts of the s levels of the Σ^-p atom, as in the case of the $p\bar{p}$ atom, the hyperfine splitting of the levels with $S = 0$ and 1 can be expected to be significant—of the order of the shift itself.

3. Equation (1) also determines the position of the deeper nuclear level Qs, which perturbs the Coulomb spectrum. In contrast with the pp atom,[2] however, there is a large uncertainty in the calculated position of this level; specifically, ReE_{Qs} varies from -1 to -6 MeV for the various versions of $a_{1/2}$ (Table I). In certain cases

this pole turns out to lie on a nonphysical sheet. Nevertheless, we should keep in mind the possibility that the radiative transition $np \to Qs$ could contribute to the spectrum of bremsstrahlung γ rays accompanying the decay of the $\Sigma^- p$ atom ($\Sigma^- p \to \Lambda n$). Observation of these γ rays would constitute an independent method for measuring the position of the Qs level when it lies on the physical sheet.

4. An important parameter of the problem is[7] $\xi = 1/2\pi\,\mathrm{Im}(a_B/a_{cs})$: For $0 < \xi < 1$, there is a reconstruction of the spectrum, while for $\xi > 1$ there is an oscillatory situation. For the pp atom this parameter is $\xi \approx 0.08 \ll 1$, but in the case considered in the present letter it is $\xi \approx 1$-2. The reason is the stronger absorption in the $\Sigma^- p$ atom.

Let us examine the case of Coulomb repulsion, $\zeta < 0$. As a^l_{cs} varies from $-\infty$ to 0, Eq. (1) traces out a curve on the k plane, which corresponds to a Breit-Wigner resonances: $k = k_1 + k_2, k_2 < 0$. When we single out from k_2 a factor corresponding to the penetrability of the Coulomb barrier,

$$k_2 = -\frac{6\pi\zeta^2}{k_1}\left\{\exp\left(\frac{2\pi|\zeta|}{k_1}\right) - 1\right\}^{-1} Q_l(k_1) \qquad (3)$$

we find the curves in Fig. 1 for various values of r_{cs}/a_B. The function $Q_l(k_1)$ varies smoothly and is finite at the origin.

Figure 1 shows points corresponding to subbarrier Coulomb resonances in light

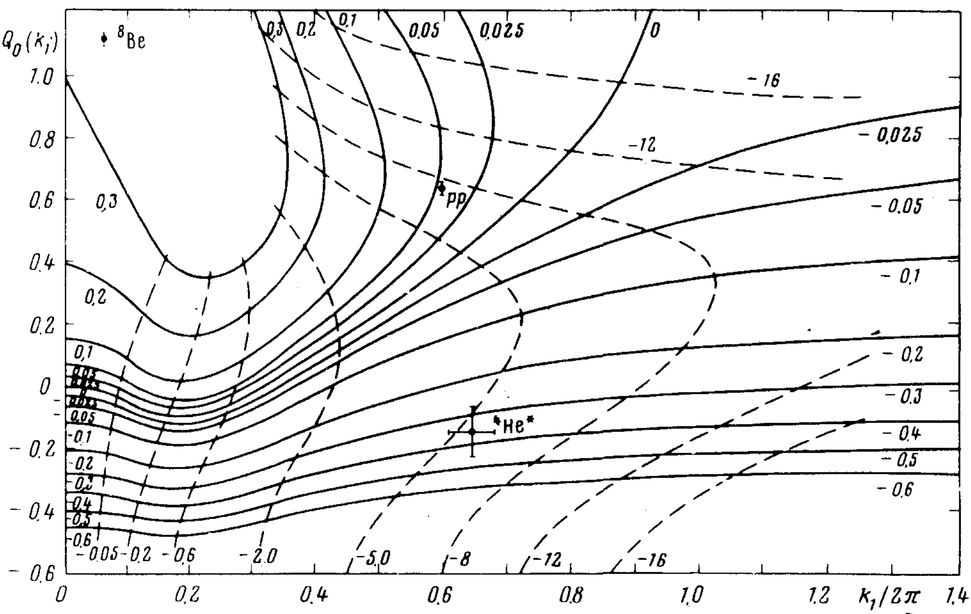

FIG. 1. Solid curves–The function $Q_l(k_1)$ for $l = 0$ and $\zeta = -1$ for various values of r_{cs}; dashed curves–lines of a_{cs} = const; curve labels–numerical values of the parameters r_{cs}/a_B and a_B/a_{cs}.

nuclei: pp scattering, the ground level of the nucleus $^8Be \rightarrow \alpha\alpha$, and the excited state of an α particle, α^* (20.1 MeV)$\rightarrow {}^3H + p$. We see that the parameters a_{cs} and r_{cs} are determined from the energy of the resonance, $E = E_0 - i\Gamma/2$. The effective radius may be negative,[3)] and it can be shown that this is a natural result in a two-channel problem with a closed channel. The situation is easily clarified for the example of α^*: The given level does not consist entirely of the decay products of $^3H + p$ and is instead formed through other channels also ($^3He + n$, $d + d$, etc).

This discussion has dealt with the case $l = 0$. The curves of $Q_l(k_1)$ for $l \geq 1$ are similar in shape. They can be used to analyze the excited levels of very light nuclei, for example, ^5Li and ^5He. These questions will be taken up in a more detailed paper.

[1)]These quantities can be related to the low-energy parameters of the strong interaction, a_s^l and r_s^l.

[2)]This will require a more detailed analysis incorporating the small amplitude $a_{3/2}$, the kinematic violation of isotopic invariance due to the difference between the masses m_Σ and m_N, etc.

[3)]For example, $r_{cs} = -0.37 a_B$ for α^*. The accurate position of the poles for the pp and ^8Be systems was recently calculated by Kok.[12] We note that the pp-scattering singularity does not lie on the imaginary axis in the k plane as a virtual nn level but is instead shifted into the complex plane because of the Coulomb cut.

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