Character of the phase transition in π condensation

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Correlation effects accompanying the formation of pion condensate are considered. An attracting long-range $\pi\pi$ interaction exists near the transition point. The condensate is the result of a first-order transition with a small jump of the meson field.

1. The possible existence of π condensate in nuclear matter was considered in [1–3]. The physical picture of the phase transition can be determined in a model with a $\pi\pi$ interaction of the $\lambda\phi^4$ type,[2] and this model is justified so long as the condensate field is weak. It turns out that even in the case of small ϕ the model with $\lambda\phi^4$ can be improved by introducing a long-range $\pi\pi$ interaction near the transition point. The amplitude of the effective 4-pion interaction then becomes an abrupt function of the coordinates and reverses sign. Since the π condensate is produced with a wave vector $k_0 \neq 0$ and the phase volume of the mesons "spoiled" by

the medium is large, it is necessary, when determining the condensate field, to take into account the virtual rescattering of the condensate mesons not only by one another but also by the mesons outside the condensate. We confine ourselves to consideration of an isotopically invariant medium with $N=Z$. We use henceforth the meson units $\hbar=\mu=c=1$.

2. The propagation function of the meson in the medium is given by

$$D^{-1}(k,\ \omega) = -1 - k^2 - \Pi(k,\ \omega) + \omega^2 . \qquad (1)$$

The proximity of the meson field to the phase transition means that the quantity $1+k^2+\Pi(k, 0)$ is small in comparison with unity at $k=k_0$, and can be expanded in powers of $k^2-k_0^2$ near the minimum; in addition, at low frequencies $\omega^2 \ll 1$, the principal dependence of D on ω is contained in Π and the term ω^2 of (1) can be neglected

$$D^{-1}(k,\ \omega) = -\omega_0^2 - \frac{\gamma(k^2-k_0^2)^2}{4k_0^2} - \Pi(k_0,\omega) + \Pi(k_0 0).$$

At $\omega<\epsilon_F$, the principal part of $\Pi(\omega)-\Pi(0)$ is due to the particle hole diagrams

$$\Pi(\omega) - \Pi(0) = -i\frac{dn}{d\epsilon_F} f^2k^2 \frac{1}{(1+g')^2}\frac{\pi|\omega|}{2kv_F^0} .$$

We can express D finally in the form

$$D(k,\omega) = -\frac{1}{\omega_0^2 + \dfrac{\gamma(k^2-k_0^2)^2}{4k_0^2} - i\chi|\omega|} \qquad (2)$$

The strong damping of the ions is due to their disintegration into particle-hole pairs; the constant χ in (2) is proportional to the adiabaticity parameter $M^2/\mu^2 \gg 1$, viz., $\chi=M^2k_0/\pi(1+g')^2$. The quantity $\omega_0^2=1+k_0^2+\Pi(k_0, 0)$ as a function of the density of nuclear matter passes through zero; $\omega_0^2=n_c-n$. In fact, D is already the propagator of a well-defined particle, and corresponds to a formation consisting of a meson and a cloud of excitations of nuclear matter. The long-range $\pi\pi$ interaction is due to two-meson exchange, the contribution of which is given by

$$L(\mathbf{k}) = \int D(\mathbf{q},\ \epsilon)D(\mathbf{k}-\mathbf{q},-\epsilon)\frac{d\epsilon dq}{(2\pi)^4} . \qquad (3)$$

As $k\to 0$, the singularities of the functions D in (3) come closer together, and this leads to divergence of $L(0)$ near the transition point: $L(0)\sim-(1/\omega_0)$. In the coordinate representation, this corresponds to a long-range action in the form

$$V(r) \sim -\frac{(\sin k_0 r)^2}{r^2}\ln\frac{\gamma}{4r^2\omega^2} \qquad r\omega_0 \ll 1,$$

$$V(r) \sim -\frac{(\sin k_0 r)^2}{r^3}\frac{1}{\omega_0}\exp -\frac{2r\omega_0}{\sqrt{\gamma}} \qquad r\omega_0 \gg 1 . \qquad (4)$$

3. The pion interaction energy takes the form

$$H_{\pi\pi} = \frac{1}{4}\int\Lambda(\mathbf{r}_1,\mathbf{r}_2,\mathbf{r}_3,\mathbf{r}_4)\phi(\mathbf{r}_1)\phi(\mathbf{r}_2)\phi(\mathbf{r}_3)\phi(\mathbf{r}_4)\,d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3 d\mathbf{r}_4) \qquad (5)$$

The total amplitude Λ includes a local part λ which does not contain two-meson diagrams: $H^0_{\pi\pi}=\frac{1}{4}\lambda\phi^4$. The long-range diagrams in Λ correspond to the case when two of the coordinates r in (5) are close, and the two others are far. There exist then for Λ three "dangerous" channels corresponding to the three possible choices of the close and far r. The contribution of the simplest diagram (3) to the energy $H_{\pi\pi}$ is equal to

$$\frac{3}{4}\lambda^2\int V(\mathbf{r}_1-\mathbf{r}_2)\phi^2(\mathbf{r}_1)\phi^2(\mathbf{r}_2)\,d\mathbf{r}_1 d\mathbf{r}_2 = \frac{3}{4}L(0)\overline{\phi^2}\,\overline{\phi^2}\lambda^2.$$

On the other hand, summation of all the "dangerous" diagrams leads to the expression

$$H^{(1)}_{\pi\pi} = \frac{3}{4}\lambda^2 L(0)\frac{\overline{\phi^2}\,\overline{\phi^2}}{1-L(0)\lambda} \qquad \lambda>0;\quad L(0)<0 . \qquad (6)$$

We consider the case of a condensate field in the form of plane layers: $\phi_0=a\sin k_0 z$

$$H_{\pi\pi} = H^0_{\pi\pi} + H^{(1)}_{\pi\pi} = \frac{a^4}{4}\frac{3}{8}\frac{\omega_0-\omega_1}{\omega_0+\omega_1} \qquad (7)$$

ω_1 is a small quantity proportional to $1/M^2$: $\omega_1=L(0)\lambda \approx 0.05\lambda$. $H_{\pi\pi}$ vanishes at $\omega_0=\omega_1$ and reverses sign with further increase of density, while the term proportional to a^2 in the energy is still positive at the same time. To find the optimal ϕ_0 at $H_{\pi\pi}<0$ it is necessary to take into account in H the next higher terms in powers of ϕ_0^2. It turns out that the principal contribution of these terms can be taken into account by replacing D in (3) with the exact function of meson propagation in the condensate field. Then the quantity ω_0 in (7) is already dependent on ϕ_0^2. We present only the final result, namely: the field ϕ_0^2 is produced jumpwise at a small value $\sim\omega_1$, and the energy gain comes into being starting with the zero value. The change in the sign of the effective energy appears to be a common property of the instability at $k\neq 0$. This was noted in [4] in the case of classical systems.

4. Near the transition point, the quasiclassical nucleon spectrum undergoes an appreciable restructuring. The one-meson diagram yields a nonanalytic contribution to the nucleon self-mass Σ

$$\Sigma_\pi = \nu[(i\chi|\epsilon| - \omega_0^2)^{1/2} - i\omega_0]\operatorname{sign}\epsilon \qquad \nu\sim 1. \qquad (8)$$

The energy ϵ_p and the damping γ_p of the quasiparticles at the Fermi surface are given by

$$\epsilon_p = \frac{v_F^0(p-p_F)}{1+\nu\chi/2\omega_0} , \qquad \gamma_p = \frac{1}{8}\nu\frac{\epsilon_p|\epsilon_p|\chi^2}{\omega_0^3} .$$

The quasiparticles vanish at $\epsilon_p\sim\omega_0^3/\nu\chi^2$, when γ_p becomes comparable with ϵ_p. The quasiparticle mass increases as $\omega_0\to 0$:

$$m^*/m \approx \nu\chi/2\omega_0 .$$

For from the Fermi surface Σ_π is independent of ω_0:

$$\Sigma_\pi = \nu(i\chi|\epsilon|)^{1/2}\mathrm{sign}\,\epsilon \qquad \chi_\epsilon >> \omega_0^2 .$$

Thus, Σ_π acquires a root singularity in ϵ as $\omega_0 \to 0$.

5. The phenomena considered above are connected with the long-range action that comes into play in the immediate vicinity of the transition point. In real nuclei, this long-range action is inevitably cut off at dimensions on the order of the radius of the nucleus. Therefore the foregoing results are fully applicable only to large systems. It can be concluded from the calculations of [1,3], however, that the nuclear matter is close to a phase transition and there are grounds for assuming that $\omega^2 \ll 1$, although $\omega_0 > \omega_L$ is also possible. Formula (4) then remains valid and the two-meson graph should be regarded as long-range, alongside the Fermi-liquid long-range graphs of the particle-hole type, and must not be regarded as a local graph with effective radius $\sim \frac{1}{2}$. We emphasize that we are dealing with long-range action over distances on the order of the nuclear dimension, and not close to unity.

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