Possibility of using anomalous passage of gamma quanta to intensify bounded beams in a gamma laser

A. V. Andreev and Yu. I. Il'inskiĭ

Moscow State University
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We consider the question of amplifying bounded wave beams by using the effect of anomalous passage of γ quanta. It is shown that the beam broadening is described approximately by an equation of parabolic type, and estimates are presented for the spreading of Gaussian beams.

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1. The possibility of using the effect of anomalous passage (EAP) of γ rays in a γ laser was investigated in detail in[1,2]. The results obtained there were based on consideration of the case of amplification of a plane wave in a semi-infinite crystal. Yet it has been emphasized many times in the literature that one of the preferred shapes of the working medium of a γ laser is that of a needle. A laser working element of this shape offers many advantages. First, since the single-pass variant of the γ laser is the most promising, a needle-shape element will ensure a sharply directional beam of stimulated coherent γ radiation.[3] Furthermore, a needle shape makes it possible to decrease the heating of the working element by the presence of the cascade γ quanta.[4] Finally, the needle shape advantage lies also in the possibility of growing relatively rapidly perfect crystals in this form (whiskers), a most important factor in the γ-laser variant in which long-lived nuclear isomers are used.

It is appropriate to raise the question of the effectiveness of the use of the EAP in the case of a needle-like configuration of the active crystal, since it may seem at first glance that the EAP can appear effectively only in samples with not too large a ratio of the longitudinal dimension to the transverse one, since the two waves existing in this case propagate relative to each other at appreciable angles (on the order of 10°).

2. The general solution of the problem of diffraction under conditions of the spatially-inhomogeneous dynamic problem was obtained in[5]. However, the integral form of the solution given there does not make it possible to obtain an analytic form of the solution even in the simplest cases, and this makes it difficult to analyze those peculiarities of the process which are of interest to us. In this paper we therefore expand the incident beam in plane waves on the entrance face, thereby reducing our problem to that previously considered in[2].

Assume for simplicity that the monochromatic γ-quantum beam incident on the crystal at the Bragg angle is bounded only in one coordinate, and that the reflecting plane of the crystal is perpendicular to the entrance face. The slowly-varying amplitudes obey inside the crystal the following system of equations (see[2]):

$$\begin{aligned} \sin\theta\frac{\partial E_0}{\partial x}+\cos\theta\frac{\partial E_0}{\partial z}&=g_{00}E_0+g_{01}E_1\\ -\sin\theta\frac{\partial E_1}{\partial x}+\cos\theta\frac{\partial E_1}{\partial z}&=g_{10}E_0+g_{11}E_1 \end{aligned},\qquad(1)$$

where θ is the Bragg diffraction angle, and E_0 are and E_1 the electric field intensities of the transmitted and diffracted waves.

With respect to the coefficients g_{ij}, it can be stated that $g_{00}=g_{11}$ in our case, and that $g_{10}=g_{01}$ in those cases when the complex polarizability satisfies the condition $\chi(\mathbf{r})=\chi(-\mathbf{r})$, satisfaction of which will henceforth be implied.

We expand the incident beam in plane waves on the entrance face, and assume furthermore that each plane wave propagates in the crystal with its own damping coefficient. Thus, the slowly varying amplitudes of the intensities of the diffracted and transmitted waves inside the crystal can be expressed in the form

$$E_\alpha(x,z)=\frac{1}{2\pi}\int_{-\infty}^{\infty}d\nu E_\alpha(\nu)\exp[i(\nu x+\mu z)],\qquad(2)$$

where $\mu\equiv\mu(\nu)$ and $\alpha=0$ or 1.

tion, we obtain the expression

$$i\mu = \cos^{-1}\theta[g_{00} \pm \sqrt{g_{01}^2 - \nu^2\sin^2\theta}]. \qquad (3)$$

The use of the inequality

$$|g_{01}| \gg \nu\sin\theta, \qquad (4)$$

which we shall show later to be well satisfied under our conditions, yields a quadratic dependence of μ on ν:

$$i\mu = \frac{g_{00} - g_{01}}{\cos\theta} + \frac{\sin^2\theta}{2g_{01}\cos\theta}\nu^2, \qquad (5)$$

meaning a transition in the (x, z) plane to the parabolic equation

$$\frac{\partial E}{\partial z} \simeq \frac{g_{00} - g_{11}}{\cos\theta}E - \frac{\sin^2\theta}{2g_{01}\cos\theta}\frac{\partial^2 E}{\partial x^2}. \qquad (6)$$

On going from Eq. (3) to Eq. (5) we have left out the plus sign in front of the square root in (3). The two signs in (3) denote that each plane wave gives rise to two modes that propagate in the crystal each with its own gain ($\mathrm{Re}(i\mu^{(1,2)})$). It can be shown that in one of the modes the amplitudes of the transmitted and diffracted waves, with our approximations taken into account, coincide, while in the other (corresponding to the minus sign in (3) they are equal in magnitude but opposite in sign. This last mode corresponds to the EAP. In Eq. (6), E can be taken to mean the amplitude of either the transmitted or diffracted wave of this mode.

We note that the parabolic equation (6) has a complex diffusion coefficient, the imaginary part of which is much larger than the real one. The beam broadening in the course of its propagation in the crystal will thus be due mainly to the transverse diffusion of the slowly varying amplitude, in analogy with the diffraction problems in quasioptics.[6]

Thus, for example, for a Gaussian beam ($E_0(x, 0)$ $= E_0\exp(-x^2/a_0^2)$), the increase of the beam radius with increasing distance from the entrance face is given by

$$a(z) = \sqrt{a_0^2 + \left(\frac{z}{a_0}B\right)^2}$$

where $B = 2\sin^2\theta(g_{01}^{11}\cos\theta)^{-1}$. For 25-keV γ quanta in the crystal lattice of aluminum we have $B = 5\times10^{-6}$ cm. Thus a beam a tenth of a millimeter wide on entry doubles in width over a distance of approximately half a meter.

An estimate of the minimal beam widths satisfying the condition (4) shows that this condition is not stringent for our purposes and for the numerical values of the energy and lattice constant indicated above it is given by $a_0 \gg 10^{-5}$ cm.

We note in conclusion that if we were to consider a beam bounded also in the second coordinate, namely, in the coordinate perpendicular to the scattering plane, then the usual diffraction of the beam would take place in this direction (and would be much weaker than the spreading in the scattering plane), since a slight perturbation of the wave vector along this coordinate would produce no new nodes on the Ewald sphere.

3. Thus our calculations allow us to conclude that the use of needle-shaped working elements does not restrict the possibility of using the EAP, since the Bragg reflection from the aggregate of the atomic planes will confine the beam to the vicinity of the sample axis over very large lengths of the latter.

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