Kinetics of parametric instability of spin waves in an antiferromagnet

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The growth of the number of magnons n_k in the course of their parametric excitation was investigated experimentally. It turned out that during the initial stage of the process n_k grows exponentially. The instability increment is smaller by a factor 2.5 than the value that follows from the simple model in which the damping is taken into account phenomenologically.

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Parametric excitation of magnons was investigated in many theoretical and experimental studies. This phenomenon is based on the interaction of a pair of magnons having oppositely directed wave vectors **k** and $-\mathbf{k}$ with a homogeneous microwave pump field **h**. When h exceeds a threshold value h_c, an exponential increase takes place in the number of magnons whose frequency is equal to half the pump frequency: $\omega_\mathbf{k}=\omega_p/2$. Equations describing the parametric instability of the magnons[1] during the linear stage of the process are analogous to the equations for the amplitude and phase of the oscillations of a parametrically excited pendulum. These equations lead to the following evolution of the process: during a time on the order of the magnon lifetime τ_M ($\tau_M \approx 1$ μsec in the investigated $CsMnF_3$ sample) the phase of all the magnon pairs adjust themselves to a value corresponding to the maximum growth rate, after which the number of magnons increase like

$$n_\mathbf{k} = n_{\mathbf{k}T} \exp\left[\left(\frac{h}{h_c} - 1\right)\frac{t}{\tau_M}\right] \tag{1}$$

where $n_{\mathbf{k}T}$ is a quantity on the order of the number of the thermal magnons with wave vectors **k** and $-\mathbf{k}$, while the time t is reckoned from the instant when the pump is turned on.

As follows from[2,3], the nonlinear effects that limit the number of parametric magnons come into play at $n_\mathbf{k}/n_{\mathbf{k}T} \approx 10^6$.

Parametric excitation of spin waves is registered in experiment by observing the absorption of the microwave pump power in a sample placed in a microwave resonator. However, the number of thermal magnons is too small for this absorption to be noticeable immediately after the pump is turned on. For the experimental installations employed in[2,3] and in the present investigation, the energy absorption becomes noticeable at $n_\mathbf{k}/n_{\mathbf{k}T} \approx 10^5$. Therefore, when the action on the sample takes the form of a rectangular pulse, the amplitude of the microwave signal $P \sim h^2$ passing through the resonator remains unchanged for a certain time, and at an instant of time t_0, when $n_\mathbf{k}$ reaches a value n^* such that the absorption of the power is fixed by the apparatus, the

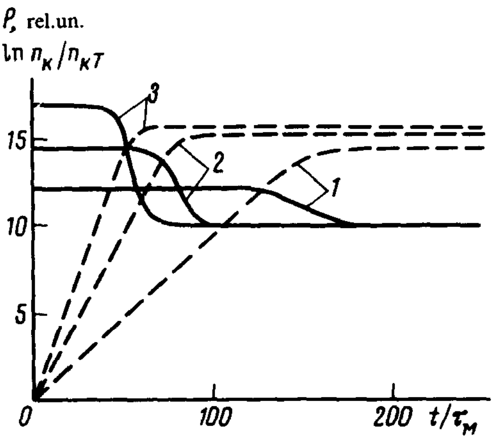

FIG. 1. Calculated form of the microwave signal passing through a resonator with a sample under parametric excitation of magnons. The nonlinear effects are not taken into account: 1—$h/h_c=1.1$; 2—$h/h_c=1.2$; 3—$h/h_c=1.3$. Dashed lines—variation of $\ln(n_k/n_{kT})$ with time.

signal decreases. Were it not for the process of nonlinear effects, the field in the resonator would decrease to the threshold value h_c, and this would serve as the reason for the limitation of the level n_k. The shape of the signal P calculated for these conditions is shown in Fig. 1 and takes the form of a step or of a pulse with a " drop". Thus, the appearance of the step is an indication that n_k has reached the level n^*. The calculation was carried out for conditions close to the experimental ones: P decreases by 10% at $n_k/n_{kT}=10^6 P$. The same characteristic shape of the transmitted signal was observed in the experiments of[2-4]. For the analysis that follows, we define n^* as the number of magnons at which the transmitted signal decreases by 2%, and define t_0 as the time of action of the pump before this decrease sets in.

In the antiferromagnets $CsMnF_3$ and $MnCO_3$ the kinetics of the process is made more complicated by the fact that τ_M increases while h_c decrease (h_c is proportional to $1/\tau_M$) with increasing n_k, i.e., a negative nonlinear damping is present.[2,3] The observed linear dependence of t_0 on $(h/h_c-1)^{-1}$, taking (1) into account, indicates that the increase of τ_M takes place either shortly prior to the instant t_0, or else after this instant, but at $0<t\lesssim 0.9t_0$ the values of τ_M and h_c remain constant. It is assumed in[2,3] that the appearance of the "drop" is due precisely to this increase of τ_M, owing to the increase of the instability growth rate $(1/\tau_M)(h/h_c-1)$. It is seen from the plots of Fig. 1 that the signal has a characteristic form of a pulse with a drop without negative nonlinear damping.

In the present paper, the rate of growth of n_k during the linear stage of the process was investigated by interrupting the pump for a time δ. After the pump was turned on, we again fixed the instant t_2 of the drop. The waveforms of the signal without and with such an action are shown in the insert of Fig. 2.

The pump was interrupted with the aid of a germanium modulator that covered the waveguide[4]; the phase of the pump was thus preserved. We investigated a $CsMnF_3$ sample, an antiferromagnet with anisotropy of the easy plane type. The pump frequency was $\omega_p/2\pi=36$ GHz, the sample temperature was $T=1.6$ K, and the external magnetic field was $H=1.2$ kOe.

If it is assumed that n_k attenuates during the time interval δ to the thermal value

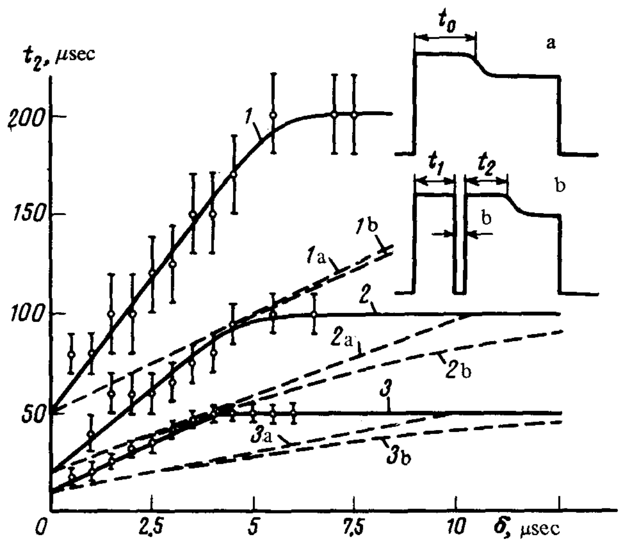

FIG. 2. Diagram of microwave signal: a—without interruption of the pump, b—with interruption of the pump for a time. Plots of $t_2(\delta)$: 1—$h/h_c=1.08\pm0.02$, $t_0=200$ μsec, $t_1=150$ μsec; 1[a]—calculation for $\tau_M=0.1$, 1[b]—$\tau_M=3$ μsec. 2—$h/h_c=1.13\pm0.02$, $t_0=100$ μsec; $t_1=80$ μsec; 2[a]—calculation for $\tau_M=0.1$, 2[b]—$\tau_M=3$ μsec. 3—$h/h_c=1.25\pm0.02$, $t_0=50$ μsec; $t_1=40$ μsec. 3[a]—calculation for $\tau_M=0.1$, 3[b]—$\tau_M=3$ μsec.

n_{kT} in accordance with the law $n_k(t)=n_{kT}+(n_k(t_1)-n_{kT})e^{-(t-t_1)/\tau_M}$, and during the remainder of the action of the pump it increases in accordance with formula (1), then from the conditions that $n_k=n^*$ at the instant of time t_0 [Fig. 2(a)] and $t_1+\delta+t_2$ [Fig. 2(b)] we obtain the $t_2(\delta)$ dependence shown for different values of h/h_c by the dashed lines in Fig. 2. The experimental data correspond to steeper $t_2(\delta)$ dependences (solid lines on Fig. 2).

The time required for n_k to reach the value n^* after the interruption depends also on the instant of interruption t_1. The $t_2(t_1)$ dependence is shown in Fig. 3. The data of Figs. 2 and 3 indicates that under conditions when $n_k \gg n_{kT}$ in the course of the damping (i.e., at values of t_1 and δ such that t_2 does not fall in the region where the plot of $t_2(t_1,\delta)$ turns into the horizontal asymptote $t_2=t_0$ the quantities t_2, t_1, and δ are connected by the relation

$$t_2 = t_0 - t_1 + \alpha\delta, \text{ where } \alpha = (2.5 \pm 0.3)/(h/h_c - 1).$$

The validity of this equation was verified for values of h/h_c in the interval 1.1–1.53. The results of experiments performed for different values of the constant magnetic field and at $T=2.1$ K are also described by this relation with the same value of α.

This means that when the pump is turned on and off in succession [Fig. 2(b)] the number of magnons n_k in the region $n_k \gg n_{kT}$ depends on the time of action of the pump $t=t_1+t_2$ and on the duration of the pause δ in the following manner:

$$n_k(t,\delta) = Af(t_1 + t_2 - \alpha\delta). \quad (2)$$

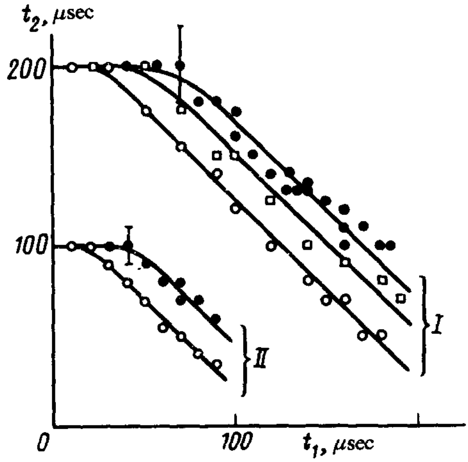

FIG. 3. Plot of $t_2(t_1)$ at fixed δ: ●—$\delta=2$ μsec; ○—$\delta=1$ μsec, □—$\delta=1.5$, μsec, I—$h/h_c=1.1\pm0.02$, $t_0=200$ μsec, II—$h/h_c=1.13\pm0.02$, $t_0=100$ μsec.

Since n_k is small enough to be able to describe the magnon damping by linear equations, and large enough in comparison with n_{kT}, then the magnon damping satisfies the law $n_k(\delta)=n_0\exp(-\delta/\tau_M)$. (This is the definition of τ_M.) On the other hand, the damping is described by the function f at $t=0$. This yields $f(x)=A\exp(x/\alpha\tau_M)$. The increase of the number of magnons is described by the function f at $\delta=0$, i.e.,

$$n_k(t) = A\exp\left\{\left(\frac{h}{h_c}-1\right)\frac{t}{(2.5\pm0.3)\tau_M}\right\} \tag{3}$$

The growth rate $\xi=(h/h_c-1)/2.5\tau_M$ of this exponential is smaller by a factor 2.5 than in formula (1).

A disparity between the real value of the growth rate ξ and the quantity $(h/h_c-1)/\tau_M$ can be expected if we assume the presence of interactions that characterize the phase of the pair of magnons without dissipation of the magnons themselves (for example, scattering by magnetic inhomogeneities[5]). Numerical simulation of the parametric instability in the presence of such interactions (the phase of the pair of magnons takes on random values after time intervals with average values τ_{ph}) has revealed two consequences of the introduction of such processes: 1) the threshold field increases: $h_c=h_{c0}F(\tau_M/\tau_{ph})>h_{c0}$; 2) the growth rate is $\xi=\beta(h/h_c-1)$, but $\beta=\beta(\tau_M/\tau_{ph})\geqslant 1/\tau_M$, meaning that ξ increases rather than decreases in comparison with $(h/h_c-1/\tau_M)$, i.e., these interaction processes cannot explain the observed discrepancy. Taking the first consequence into account, we note that on the whole, at equal values of h, the value of ξ, naturally, decreases in comparison with that in a magnetic system in which there are no processes that randomize the phase.

It has thus been established experimentally that the evolution of the parametric instability is exponential in time, but the growth rate differs from that predicted by the simple model (formula 1) and by the model that takes into account the singularities of scattering by magnetic inhomogeneities.

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