Influence of polarization of vacuum in a magnetic field on the propagation of radiation in a plasma

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It is shown that the polarization of vacuum by a strong magnetic field leads to a substantial change in the polarization, in the angular distribution, and in the emission spectrum of a magnetoactive plasma.

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In the presence of a strong magnetic field, the electron-positron vacuum behaves like an anistropic medium with birefringent properties (polarization of the vacuum by a magnetic field).[1–4] Novik *et al.*[5] have called attention to the need for taking this effect into account when considering the polarization of x rays from a neutron star even in the case when the magnetic field is $B \ll B_c = m_e c^3/e\hbar = 4.41 \times 10^{13}$ G (i.e., $\hbar\omega_B \ll m_e c^2$). However, the main conclusion of [5], that the radiation becomes depolarized when propagating in a magnetized vacuum, turned out to be incorrect.[6]

In this paper we consider the influence of polarization of vacuum in a strong magnetic field on the propagation of radiation in a magnetoactive plasma. We shall show that the polarizations and the angular distribution of the emission of a "plasma + vacuum in a magnetic field" system can differ substantially, at definite relations between the concentration N_e and the magnetic field B, from those in the case of an ordinary magnetoactive plasma. The main cause for this difference in the different character of the polarization at the normal (natural) waves in a magnetoactive plasma and in a magnetized vacuum: in vacuum, the normal waves are linearly polarized for the entire range of angles θ between the propagation directions of the radiation and of the magnetic field,[2] while in a magnetoactive plasma they are elliptically polarized.[7] At equal degrees of influence of the vacuum and of the plasma on the polarization and on the coefficients of the normal waves, specific effects appear in the polarization, angular distribution, and the emission spectrum of such a system.

The Lagrangian of the "plasma + vacuum" system in a constant magnetic field **B** and in the wave field (**E**, **B**′) at $\mathscr{B} \ll B_c$, $\hbar\omega \ll m_e c^2$ and $V \ll c$ can be written in the form

$$L = \frac{1}{8\pi}(E^2 - \mathcal{B}^2) + \frac{2\pi}{45(4\pi)^2 B_c^2}\left[(E^2 - \mathcal{B}^2)^2 + 7(\mathbf{E}\mathbf{B})^2\right] + \frac{1}{c}\,\mathbf{j}\,\mathbf{A}, \tag{1}$$

[wh]ere $\alpha = e^2/\hbar c$, A is the vector potential of the radiation field, $\mathbf{j} = -eN_e\mathbf{V}$ is the [cur]rent density, and $B = B + B'$. The electron velocity **V** is obtained by solving the [no]nrelativistic equation of motion of the electron in the wave field (**E**, **B**′) and in the [ext]ernal magnetic field **B**, with allowance for the damping; the solution of this equa-

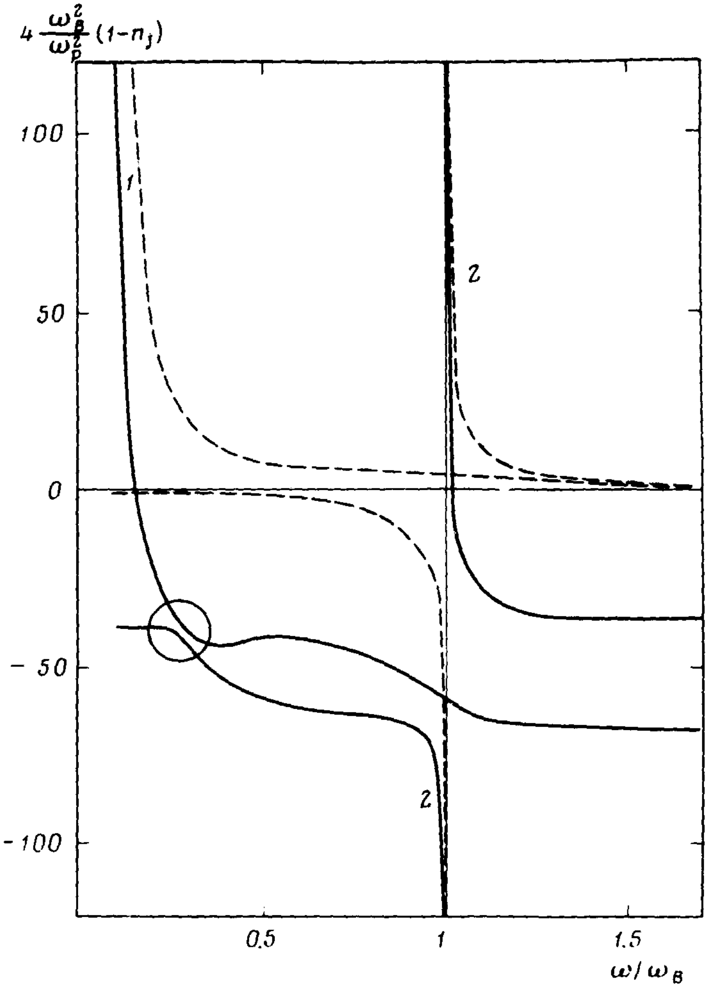

FIG. 1. Dispersion curves $4(1-n_j(\omega))(\omega_B^2/\omega_p^2)$ at $\theta=80°(j=1.2)$ for a magnetoactive plasma with ($N_e=0.6N_m$) and without (dashed curves) allowance for the polarization of the vacuum. The circle marks the region of approach of the dispersion curves, where linear transformation of various modes in an inhomogeneous plasma is possible.

tion is well known. By differentiating (1) with respect to **E** and **B′** we obtain th electric induction **D** and the magnetic field intensity **H**. At $E \ll B$ and $B' \ll B$, the term quadratic in E and B' can be discarded. Defining in the usual manner the permittivit and the magnetic permeability tensors, we easily write down a dispersion equatio whose solutions are the refractive indices of the normal waves propagating in tl system under consideration. For a tenuous plasma ($v=\frac{\omega_p^2}{\omega^2}=\frac{4\pi e^2 N_e}{m_e\omega^2}\ll 1$) th take the form

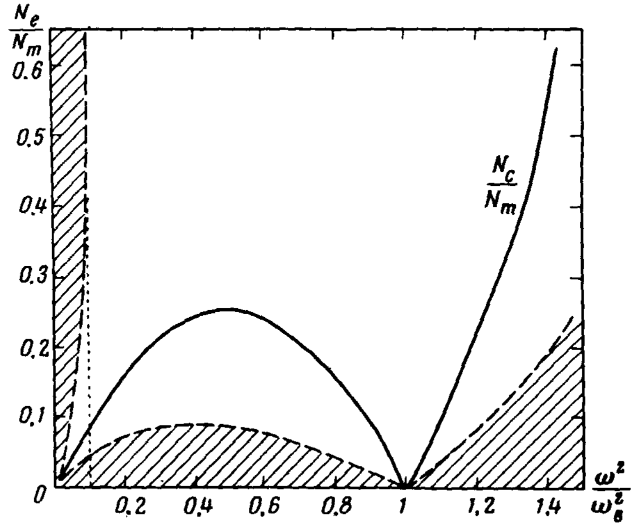

FIG. 2. Character of the polarization of the normal waves in a magnetoactive plasma with allowance for the polarizability of the vacuum by the magnetic field, as a function of the plasma concentration N_e and of the emission frequency ω. The dashed line separates the regions of predominantly linear ($|q(\theta=45°)|>1$, shaded) and circular ($|q(\theta=45°)|<1$ polarization of the normal waves. The dotted straight line separates the corresponding regions without allowance for the vacuum.

$$(n+i\kappa)_{1,2}=1-\frac{1}{4}\left\{v\left[\frac{\sin^2\theta}{1-i\gamma}+\frac{(1+\cos^2\theta)(1+i\gamma)}{1-u+2i\gamma}\right]-11a\sin^2\theta\right.$$

$$\left.\pm\left[\left(\frac{vu}{1-u+2i\gamma}+3a\right)^2\sin^4\theta+\left(\frac{2v\sqrt{u}\cos\theta}{1-u+2i\gamma}\right)^2\right]^{1/2}\right\}, \qquad (2)$$

where

$$u=(\omega_B^2/\omega^2),\quad a=\frac{e^2}{\hbar c}\,\frac{1}{45\pi}\left(\frac{B}{B_c}\right)^2,\quad \gamma=(\nu/\omega)\ll 1,$$

nd ν is the effective collision frequency. The real parts of the refractive indices $n_{1,2}$ of he normal waves are shown in Fig. 1. The character of the polarization of the normal ʼave was determined by the real part of the ratio of the terms in the second square racket of (2):

$$q=q_0\left\{1+\frac{3a(1-u)}{vu}\right\}=q_0\left\{1-\frac{N_c}{N_e}\,\mathrm{sign}(u-1)\right\};\quad q_0=\frac{\sqrt{u}\sin^2\theta}{2\cos\theta};$$

$$N_c=N_m\frac{|1-u|}{u^2};\quad N_m=\frac{1}{60\pi^2}\left(\frac{m_e c}{\hbar}\right)^3\left(\frac{\hbar\omega_B}{m_e c^2}\right)^4=4.5\times10^{28}\left(\frac{B}{B_c}\right)^4. \qquad (3)$$

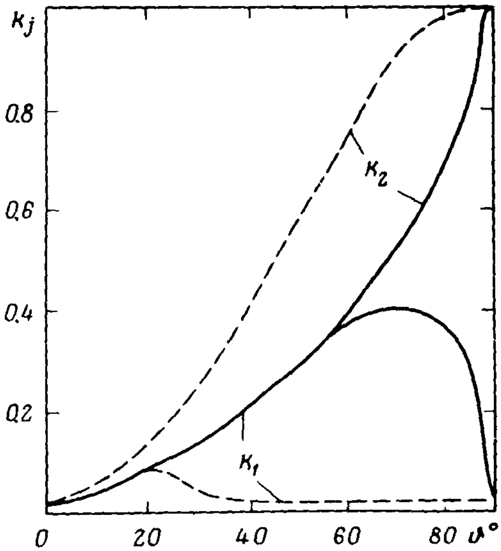

FIG. 3. Angular dependence of the absorption coefficients of the normal waves $k_j = 2(\omega/c)\kappa_j$ without (dashed) and with allowance for the polarization of the vacuum at $\omega_B/\omega = 10$ and $N_c/N_e = 0.9$.

The normal waves are polarized linearly at $|q| \gg 1$ and circularly at $|q| \ll 1$. The polarization of the normal waves is determined at $N_e \ll N_c$ by the vacuum ($|q| \gg 1$) and at $N_e \gg N_c$ by the plasma, and in the latter case at $\sqrt{u} \gg 1$ the normal waves are linearly polarized in a wide range of angles (quasitransverse propagation), while at $\sqrt{u} \ll 1$ they are circularly polarized (quasilongitudinal propagation). The solid line in Fig. 2 shows the dependence of the ratio N_c/N_m on $u^{-1} = \omega^2/\omega_B^2$.

An analysis of formulas (2) and (3) lead to the following conclusions: 1. Allowance for the polarization of the vacuum changes qualitatively the form of the refractive indices of a magnetoactive plasma. 2. At $\omega \gg \omega_B$ allowance for the polarization of the vacuum narrows down the quasilongitudinal propagation interval within which the normal waves have a circular polarization. For example, for $B = 4.4 \times 10^9$ G, $\hbar\omega = 260$ eV, and $N_e = 10^{14}$ cm^{-3}, the quasilongitudinality condition is satisfied at $\theta < 33°$ in place of $\theta < 84°$ if no account is taken of the polarization of the vacuum. A[illegible] $\omega < \omega_B$, in the frequency region $\omega \approx \omega_B(N_e/N_m)^{1/2}$, we have $|q| < 1$ for almost all θ i.e., the normal waves are circularly polarized, in full contradiction to the result[illegible] obtained without account of the polarization of the vacuum or in a vacuum without [illegible] plasma. The reason is that at $u > 1$ the plasma and the vacuum "tend" to polarize eac[illegible] normal wave linearly, but in mutually perpendicular directions (the minus sign in (3 at $u > 1$). At $N_e \sim N_c$ the influence of the vacuum and of the plasma on the linea[illegible] polarization cancel each other and the resultant polarization becomes circular ([illegible] $\theta \neq \pi/2$). 4. Since the absorption and emission coefficients of the normal waves in a[illegible] anisotropic medium depend essentially on the character of the polarization, the chan[illegible] of the polarization of q as a result of the influence of the vacuum means a substanti[illegible] change, for example, in the angular dependence of the absorption coefficient (see Fi[illegible] 3). 5. A substantial change takes place in the polarization of the cyclotron li[illegible] ($\omega \approx \omega_B$) of the emission of an optically thin plasma. Allowance for the polarizati[illegible] of the vacuum narrows down the region of angles in which the radiation is circularly polarized. The degree of linear polarization $p_e = q/(1+q^2)^{1/2}$, in contrast to the usual situation, depends on the plasma concentration.

We note in conclusion that even at a negligible difference between the refractive indices of the vacuum in the magnetic field from unity (at $\hbar\omega \ll m_e c^2$) the influence of the vacuum leads to fully observable effects, for example, in cosmic plasma.

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