Stationary levels of the electron in a field of a polarized neutral particle

M. S. Khaĭkin

Institute of Physics Problems, USSR Academy of Sciences

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It is shown that stationary "dielectric levels" of electrons near a dielectric particle exist and arise in the potential well formed by the attraction of the particle polarized by the electron field and by the work of the penetration of the electron into its material. The dependence of the spectrum of the surface levels on the curvature of the dielectric surface is obtained. The frequency spectra of the dielectric levels extend from the optical band (for the ion) to $\sim 10^{12}$–10^{11} Hz. The possible existence of dielectric levels of electrons near particles of cosmic dust is discussed.

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The attraction of an electron by a flat surface of a polarizable dielectric, under conditions when the work of penetration of the electron into the dielectric is large, forms a potential well in which stationary surface levels of the electrons are produced: such levels near the surface of liquid helium have been well investigated.[1,2] The two-dimensional layer of surface electrons situated inside a bubble in liquid (superfluid) helium ensures the stable existence of such a formation—the bubblon.[3,4] In a many-electron bubblon at sufficiently low temperatures, Wigner crystallization of the electron layers should take place; a bubblon with a small number of electrons is apparently a three-dimensional quantum system.

In this communication we consider the geometrically opposite situation, namely: the influence of the curvature of a convex surface of a dielectric on the spectrum of the surface levels of the electrons and on stationary electrons levels near small (spherical) dielectric particles. The possible existence of such quantum systems—"dielectric ions"—has a neutron polyatomic core that retains the electron by polarization forces is undisputed for substances and atoms (or molecules) capable of forming negative ions (for H^-, for example, the binding energy of the excess electron is ≈ 0.7 eV). The spectrum of the stationary levels of the electrons near a convex dielectric surface begins to differ noticably from the spectrum of a planar system (frequency of the transition from the ground state to the continuum $\nu_1 \approx 5\times 10^{12}$ Hz for solid H when the surface curvature radius R decreases to a value of the order of the distance $d_{\rm fl} \approx 10^{-6}$ cm from the flat surface to the electron.

On the other hand, the case of an extremely small particle that holds the electron by polarization forces is well known: this is a negative ion whose spectrum lies in the optical region ($\sim 10^{15}$ Hz) and having values R and d of the order of 10^{-8} cm. Thus, our problem is to determine the spectrum of dielectric ions with dimensions ranging from atomic to $\sim 10^{-6}$ cm. The relative level width will be assumed small, as is the case for stationary states over a flat surface of liquid He[5]; this is all the more true for ions (the temperature is assumed not to exceed several degrees).

It is shown in[6] that an atom polarizability that agrees well with spectroscopic measurements can be calculated (the same result is obtained also by a quantum-mechanical calculation)[7] on the basis of a simple model: the field of an electron e/r^2 located at a distance r from the center of an atom with polarizability α_A produces in the latter a dipole moment $\alpha_A e/r^2$. The potential energy of the electron in the field of this dipole is $U = -\alpha_A e^2/2r^4$. We use the very same formula for a spherical particle of radius R, containing N atoms with polarizability $\alpha_R = N\alpha_A$ (assuming its polarization to be homogeneous).

Quantization in the sense of Bohr-Sommerfeld leads to energy levels $E_n = -n^4 e^2 r_B^2/2\alpha_R$ for the electrons revolving around the small particle on orbits with radii $r_n = n^{-1}\sqrt{\alpha_R/r_B}$; ($r_B$ is the Bohr radius). The realized levels are selected by the condition $r_n \gtrsim R$, which admits for the cases of practical interest (H, He) of the single value $n = 1$.

The obtained spectrum is obviously incorrect for the region $R \gtrsim d_{\rm fl}$, since it does not go over as $R \to \infty$ into the known spectrum of the flat levels. We shall therefore obtain the spectrum for large R in a different manner.

The electron-level spectrum near a flat surface[1] is determined by a potential energy $U_{\rm fl} = -\pi\alpha e^2/2d_{\rm fl}$ (where $\alpha = (\epsilon - 1)/4\pi$ is the polarizability of the dielectric). On going to a spherical surface the energy decreases: $U(R) = U_{\rm fl} f(R/d)$, where $f(R/d) < 1$ is calculated in simple fashion at $\epsilon - 1 \ll 1$, which is close enough for H and He. Calculation yields $U(R \approx 5d_{\rm fl}) \approx 0.5 V_{\rm fl}$; the electron levels vary in the same fashion.

We turn now to estimates for solid hydrogen, based on the known data: $\alpha_A = 0.41\times 10^{-24}$ cm^3 [7]; $\alpha = 0.02$, $d_{\rm fl} = 1.7\times 10^{-7}$ cm.[2] Figure 1 shows the values of the frequncies $\nu_1(R)$ of the transition from the ground level to the continuum, calculated for the two regions $R < d_{\rm fl}$ and $R > d_{\rm fl}$ considered above (solid lines). The extreme values $\nu_1(R \sim 10^{-8}$ cm) $\sim 10^{15}$ Hz and $\nu_1(R = \infty) \sim 10^{12}$ Hz correspond respectively to the optical spectrum of the ion and to the value expected for the surface of solid

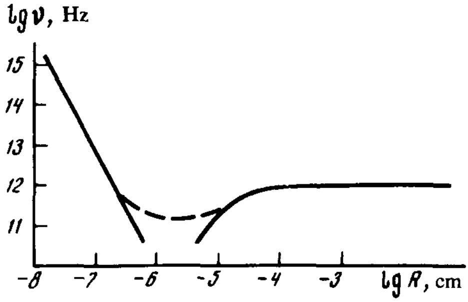

FIG. 1.

hydrogen on the basis of the experimental data for helium. In the transition region $R \sim d_{\mathrm{fl}}$, the two solutions for $\nu_1(R)$ intersect; obviously, the only levels that can be realized are those corresponding to the upper branches of the $\nu_1(R)$ curves, since they correspond to deeper potential wells. The real $\nu_1(R \sim d)$ dependence should have approximately the form of the dashed curve and lie in the interval 10^{12}–10^{11} Hz.

Let us examine the possibility of observing the dielectric levels. We must first point out the possible existence of dielectric ions in the cosmic medium, which is made up $\sim 90\%$ of H (and $\sim 10\%$ of He—in terms of the number of atoms) and which contains cosmic dust with particle dimensions from atomic to $\sim 10^{-4}$ cm. The presence of dielectric levels should be observed by the emission (and absorption) in the range from optical frequencies to 10^{12}–10^{11} Hz, with a maximum in the region of the low-frequency limit due to the extremum and the horizontal branch of the $\nu_1(R)$ curve (see Fig. 1). Of course, this region should be smeared out because of the differences between the shapes and polarizabilities (material) of the dust particles. This picture agrees fully with the existing cosmic radiation in a broad band in the vicinity of $\sim 10^{12}$ Hz, which is particularly intense from dust clouds, and whose origin has not yet been explained.[8] Obviously, a detailed analysis of the possible connection between this radiation and the dielectric level is necessary.

The organization of a laboratory experiment aimed at observing dielectric levels near small particles is undoubtedly worthy of attention; in this case the particles should be microdrops (fog) of helium, and the observation method should be the measurement of absorption in the characteristic frequency region.

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