[3] V.N. Oraevskii, Nuclear Fusion 4, 293 (1963).
[4] V.F. Aleksin and V.S. Khodusov, Ukr. Fiz. Zh. 15, 1021 (1970).

SELF-FOCUSING OF A VECTOR FIELD

V.N. Eleonskii and V.P. Silin
P.N. Lebedev Physics Institute, USSR Academy of Sciences
Submitted 11 December 1970
ZhETF Pis. Red. 13, No. 3, 167 - 170 (5 February 1971)

Volkov [1], using plasma as an example, has shown in 1958 that nonlinear action of an electromagnetic field on the medium can give rise to spatially-localized distributions of the field, which subsequently were called self-trapping or self-focusing. The nonlinear solution obtained by Volkov for the field equation pertains to the case of a scalar field. In this communication we show that self-focused field distributions exist also for TM waves, when there are two components of the electric field (the vector case). However, such distributions differ qualitatively from the case of a scalar field.

For an electric field in the form

$$\mathbf{E}(r, t) = \mathbf{E}^{+}(r)\cos\omega t + \mathbf{E}^{-}(r)\sin\omega t$$

we write down (compare with [2]) the following equations of nonlinear electrodynamics

$$-\Delta\mathbf{E}^{\pm} + \mathrm{grad}\,\mathrm{div}\,\mathbf{E}^{\pm} = k^2 \epsilon \mathbf{E}^{\pm},$$

where

$$k^2 = (\omega/c)^2, \quad \epsilon = \epsilon[\omega, (\mathbf{E}^{+})^2 + (\mathbf{E}^{-})^2].$$

For the case, which does not violate the generality, when there is no field energy flux along the x axis, assuming

$$E^{+}_{x,z} + iE^{-}_{x,z} = E_{x,z}(x)\exp(ik_z z + i\delta_{x,z}),$$

taking $\delta_z - \delta_x = \pi/2$ and assuming δ_z and δ_x to be constant, we have therefore

$$E_z'' - k_z E_x' + k^2 \epsilon[\omega, E_x^2 + E_z^2] E_z = 0, \tag{1}$$

$$k_z E_z' - k_z^2 E_x + k^2 \epsilon[\omega, E_x^2 + E_z^2] E_x = 0. \tag{2}$$

Let us consider first the solutions of the system (1) - (2) in the limit of $k_z = 0$. Then, obviously, two different situations are possible. In the first case, the longitudinal component of the electric field is $E_x = 0$, and a solution of the Volkov type is obtained for E_z. In particular, when

$$\epsilon = \epsilon_0 + \Delta(E_x^2 + E_z^2), \quad \Delta > 0, \quad \epsilon_0 < 0 \tag{3}$$

we have [3]

$$E_z = \sqrt{-\frac{2\epsilon_0}{\Delta}} / \mathrm{ch}[\sqrt{-\epsilon_0}\, k(x - x_0)], \quad E_x = 0. \tag{4}$$

The other solution of this system (1) - (2) corresponds to the presence of a longitudinal electric field $E_x \neq 0$. This is possible if

$$\epsilon[\omega, E_x^2 + E_z^2] = 0, \tag{5}$$

which in a certain sense is similar to the condition for the existence of longitudinal waves in linear electrodynamics.

We shall continue the discussion with the nonlinear dielectric constant (3) as an example. We note that at infinity, where there is no field, we have $\varepsilon = \varepsilon_0 < 0$. On the other hand, at the point of the field maximum (4) we have $\varepsilon > 0$. Therefore the solution of (4) at the point

$$x - x_0 = \ln(1 + \sqrt{2})/k\sqrt{-\varepsilon_0}$$

satisfies the condition (5) for the existence of a longitudinal field. This can cause the solution (4) to go over at small $x - x_0$ into another solution connected with the existence of a longitudinal field. To make the realization of such a possibility more obvious, we use in place of the system (1) - (2) the first integral that follows from this system and satisfies the boundary condition that the field vanish, as well as Eq. (2), which takes the form

$$(E_x')^2 - k_z^2 E_x^2 + k^2 [\varepsilon_0 (E_x^2 + E_z^2) + \frac{1}{2}\Delta(E_x^2 + E_z^2)^2] = 0, \qquad (6)$$

$$k_z E_z' - k_z^2 E_x + k^2 [\varepsilon_0 + \Delta(E_x^2 + E_z^2)] E_x = 0. \qquad (7)$$

It follows therefore that there exists a solution

$$E_z = \pm\varepsilon_0 kx/\sqrt{2\Delta}, \quad E_x^2 + E_z^2 = -\varepsilon_0/\Delta .$$

This solution is possible only for finite x. More accurately speaking, it is possible only if it goes over into (4) at large values of x. The point of such a transition is determined by $E_x = 0$, i.e., $x = \sqrt{2}/k\sqrt{-\varepsilon_0}$.

Obviously, the discussed transition of the solutions is possible if

$$x_0 = [\sqrt{2} - \ln(1 + \sqrt{2})]/k\sqrt{-\varepsilon_0} .$$

Figure 1 shows the three-dimensional phase curve constructed for $k_z/k \ll \sqrt{-\varepsilon_0}$. The necessity of the indicated transition can be verified by constructing the projections of the three-dimensional phase curve on the planes (E_z', E_z), (E_x, E_z), and (E_x, E_z').

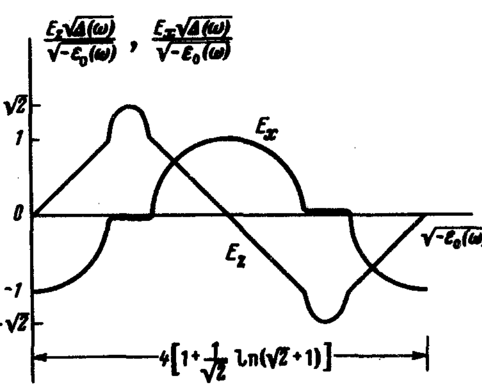

Fig. 1

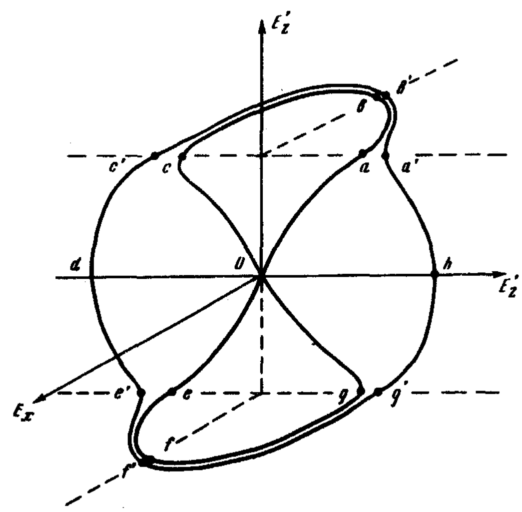

Fig. 2

An analysis has shown that there exist two phase trajectories, one of which (0abc0) corresponds to the possible existence of a spatially localized solution, which has in the limit of small k_z/k the form

$$E_z = \frac{|\epsilon_0|}{\sqrt{2\Delta}} kx, \quad E_x^2 = -\frac{\epsilon_0}{\Delta}[1 + \frac{1}{2}\epsilon_0 (kx)^2], \quad x \leqslant \frac{\sqrt{2}}{k\sqrt{-\epsilon_0}};$$

$$E_z = \sqrt{-\frac{2\epsilon_0}{\Delta}} \operatorname{sign} x / \operatorname{ch}[\sqrt{-\epsilon_0}\, k\, |x| - \sqrt{2} + \ln(1 + \sqrt{2})], \quad E_x = 0,$$

$$x \geqslant \frac{\sqrt{2}}{k\sqrt{-\epsilon_0}},$$

and the other corresponds to a spatially periodic solution with a period $4[\sqrt{2} + \ln(1 + \sqrt{2})]/k\sqrt{-\varepsilon_0}$ (Fig. 2). We emphasize that the presence of a small k_z decreases the maximum value of the self-focused field by a factor $\sqrt{2}$ compared with the solution of the scalar-field theory. In the case of a spatially periodic solution, there is alternation of the regions of the transverse field with the regions in which there is a longitudinal field. As follows from Fig. 2, the vector of the electric field rotates in this case.

[1] T.F. Volkov, Fizika plazmy i problema uprovlyaemykh termoyadernykh reaktsii (Plasma Physics and the Problem of Controlled Thermonuclear Reactions), AN SSSR 3, 336 (1958).

[2] V.P. Silin, Zh. Eksp. Teor. Fiz. 53, 1662 (1967) [Sov. Phys.-JETP 26, 955 (1968)].

[3] S.A. Akhmanov, A.P. Sukhorukov, and R.V. Khokhlov, Usp. Fiz. Nauk 93, 19 (1967) [Sov. Phys.-Usp. 10, 609 (1968)].

CONCERNING THE NATURE OF THE NONLINEARITY OF THE NaCl CRYSTAL

M.S. Brodin and A.M. Kamuz
Physics Institute, Ukrainian Academy of Sciences

Submitted 21 December 1970
ZhETF Pis. Red. 13, No. 3, 170 - 173 (5 February 1971)

The nonlinearities of condensed media, which lead to self-action of intense laser radiation in them, are usually attributed to the following mechanisms: the Kerr effect [1], striction [2], thermal [3], and SRS [4]. We wish to show that in the case of alkali-halide crystals such as NaCl there is one more possibility, which plays an important role in self-action processes. The presence of such processes (self-focusing, self-bending) in NaCl crystals is evidenced by the results of our earlier investigations [5, 6].

Back in [7] we have already shown that color centers appear in the region where the NaCl crystal is damaged by ruby-laser radiation. They correspond to absorption bands located in the visible region of the spectrum, on the side of the laser-emission line, and leading to an increase of the dispersion at the frequency of this line. This raises naturally the question of whether these centers appear already during the action of the laser pulse, and consequently have a bearing on the nonlinearity, as was assumed by us earlier [8], or whether they arise after the action of the laser pulse and have no direct bearing on the self-action processes.

To answer this question it was necessary to obtain the absorption spectrum of the NaCl crystal during the time of action of the laser pulse ($\sim 10^{-8}$