[1] U. Amaldi et al., Phys. Lett. 43B, 231 (1973); 44B, 112 (1973); S. R. Amendolia et al., Phys. Lett. 44B, 119 (1973).
[2] G. Bellettini et al., Phys. Lett. 14, 164 (1965); K. J. Folet et al., Phys. Rev. Lett. 19, 857 (1967); S. P. Denisov et al., Phys. Lett. 36B, 415 (1971).
[3] V. Bartenev et al., Phys. Rev. Lett. 29, 1755 (1972); J. W. Chapman et al., Phys. Rev. Lett. 29, 1686 (1972); G. Charlton et al., Phys. Rev. Lett. 29, 515 (1972); E. T. Dao et al., Phys. Rev. Lett. 29, 1627 (1972).
[4] G. G. Beznogikh et al., Phys. Lett. 39B, 411 (1972); Kh. M. Chernev et al., Phys. Lett. 36B, 266 (1971)
[5] G. G. Beznogikh et al., Phys. Lett. 30B, 274 (1969).
[6] U. Amaldi et al., Phys. Lett. 36B, 504 (1971); G. Barbellini et al., Phys. Lett. 39B, 663 (1972).
[7] V. A. Tsarev, Kratkie soobshcheniya po fizike (FIAN), No. 8 (1973).
[8] V. A. Tsarev, Yad. Riz. 18, 425 (1973) [Sov. J. Nuc. Phys. 18, No. 2 (1974)]; FIAN Preprint No. 69, 1973.
[9] Yu. P. Gorin et al., Yad. Fiz. 17, 309 (1973) [Sov. J. Nuc. Phys. 17, No. 2 (1973)].
[10] V. N. Bolotov et al., Report at XVI Inter. Confer. on High Energy Physics, Batavia, 1972.
[11] N. I. Starkov, V. A. Tsarev, N. P. Zotov, and P. N. Lebedev, Physical Institute Preprint No. 103, 1973.

INELASTIC INTERACTIONS OF FAST HADRONS WITH NUCLEI

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Submitted 13 August 1973
ZhETF Pis. Red. 18, No. 7, 465 - 469 (5 October 1973)

The picture of the inelastic processes accompanying the interaction of a fast hadron with a nucleus is considered under the assumption that the interaction of two hadrons is described by means of a pomeron exchange and that the asymptotic total cross sections are constant in this case.

1. The physical picture of the interaction can be easily understood by starting from the parton conceptions [1, 2] of the spatial structure of the wave function of a fast hadron of energy E, namely, its stationary state constitutes in the mean an aggregate of particles (partons) with energy spectrum $d\varepsilon/\varepsilon$ $(m < \varepsilon < E)$ and with bounded transverse momenta; the particles with energies $\sim \varepsilon$ are distributed in a disk of radius $\sim \sqrt{2\alpha' \ln(E/\varepsilon)}$ and thickness $\sim 1/\varepsilon$. When such a "comb" of partons is incident on a hadron at rest, then the hadron can interact only with a slow particle from the comb; after the interaction, which destroys the coherence of the system, the remaining particles of the comb also become free (after times on the order of $\varepsilon\mu^{-2}$), with the shape of the $d\varepsilon/\varepsilon$ spectrum remaining the same. The total cross section σ of this process is determined by the interaction of the slow particles, and therefore $\sigma \approx \mu^{-2}$ (μ is the meson mass).

We apply these conceptions to a nucleus consisting of A nucleons (the radius of the nucleus is R, the average distance between nucleons is $r \sim 1/\mu$, and the nucleon mass is m). It is convenient to consider the hadron-nucleus interaction in a coordinate system in which the nucleus has an energy AE and the hadron is at rest. What is the parton state in such a system? We assume at first that E is relatively small, and then start to increase it. Owing to the Lorentz contraction, the nucleus flattens gradually with increasing E, and the average longitudinal distances between the nucleons decrease like $(r/\sqrt{3})(m/E)$, while the average transverse distances remain unchanged.

Each of the nucleons of the nucleus is accompanied by a comb of partons. How do these combs interact? We consider two nucleons located a distance $[x_z, x_\perp]$ apart in the rest system of the nucleus. Obviously, their parton clouds are spatially separated if at least one of the conditions

$$x_\perp^2 > 8\alpha' \ln(E/m), \qquad (m/E)\, x_z > 1/\mu . \tag{1}$$

is satisfied. If $x_\perp^2 \sim 8\alpha' \ln(E/M)$, the regions in which the slow partons are situated begin to overlap transversely, and a longitudinal overlap sets in at $x_z \sim E/m\mu$. When two parton clouds begin to overlap spatially, coalescence of the combs becomes probable, and with increase of E the system will evolve as a single parton comb; the probability of such a coalescence tends to unity with increasing E (see [3]).

Let us increase the energy from $E \sim m$ to $E \geq Rm\mu$. The nucleon combs situated in tubes with approximate cross section $\sim \sigma$ begin to coalesce[1], so that at $E \sim Rm\mu$ they join into groups containing on the average $\nu \simeq A\sigma/\sigma_{in}(A) \sim A^{1/3}$ nucleons each, where $\sigma_{in}(A)$ is the total inelastic cross section per nucleus ($\sigma_{in}(A) \simeq \pi R^2 b$, with $b \leq 1$ for large A). At $E \geq Rm\mu$, the region in which the slow ends of the combs are located becomes stabilized in the form of a disk of radius R and thickness $\sim 1/\mu$; with further increase of the energy this region remains practically unchanged, only the number of slow partons is gradually decreased as a result of the increase of the Regge radii and the corresponding increase in the number of coalescing combs. The immobile hadrons on which the nucleus is incident "sees" just this disk of slow partons. The total cross section $\sigma_{in}(A, \xi)$ therefore decreases monotonically:

$$\sigma_{in}(A, \xi) \sim \pi R^2 b \left[1 + \frac{2\pi\alpha'\xi}{\sigma}\right]^{-1}, \qquad \xi = \ln(E/m) \tag{2}$$

When a slow parton collides with the hadron at $E \gtrsim Rm$, particles made up of $\sim \nu$ linked combs are produced in the final state. The total nuclear spectrum is therefore close in shape to the spectrum produced in the case of interaction of two nuclei of energy E, but the density of the produced particles is ν times larger. This leads to the following expression for the inclusive cross section at the nucleus and at average multiplicity:

$$F(A, p) = \nu \frac{\sigma_{in}(A)}{\sigma} f(p) = Af(p), \quad F(A, p) \equiv \epsilon\left(\frac{\partial^3\sigma}{\partial p^3}\right)_A ; \tag{3a}$$

$$n(A) \simeq \nu\bar{n} = A\frac{\sigma}{\sigma_{in}(A)}\bar{n}, \tag{3b}$$

where $f(p) \equiv \varepsilon(\partial^3\sigma/\partial p^3)$ and $\bar{n}$ are the inclusive cross section and the average multiplicity for a nucleon-hadron collision at energy E. Of course, (3a) no longer holds in the lower part of the spectrum, since the hadron collides there with approximately one parton, where F(A, p) should approach the spectrum per nucleus:

$$F(A,p) \simeq \frac{\sigma_{in}(A)}{\sigma} f(p). \tag{4}$$

With further increase of the energy to $E \gg Rm\mu$, relations (3a) and (4) remain valid near the end points of the spectrum. In the fast part of the spectrum, the density of the produced particles increases like $\nu[1 + (2\alpha^1\xi/r^2)]$, which is of the order of the number of linked combs, but the decrease of σ_{in} compensates for this effect, and therefore (3a) holds. Closer to the lower edge of the spectrum, F(A, p) takes the form (4), for when $E \gg Rm\mu$ the target collides only with the end of one of the combs. The intermediate region of the spectrum, in which the coalescence of the combs takes place, becomes longer. The quantity $n(A, \xi)$ increases like

$$n(A, \xi) \simeq \gamma\left[\frac{2\pi\alpha'\xi}{\sigma}\ln\xi + \nu\left(1 + \frac{2\pi\alpha'\xi}{\sigma}\right)\ln R\mu\right], \quad E \gg Rm\mu , \tag{5}$$

where $\gamma \sim 1$ and depends on the probability of the linking of the parton combs (γ is connected with the values of the "internal" pomeron vertices [3]).

Finally, a few words concerning the region of purely theoretical interest, $2\alpha^1\xi \sim R^2$, where the Regge radius becomes comparable with the radius of the nucleus and continues to grow further [4, 1, 3]. We see that in this case there remains in the disk approximately one slow particle, since almost all A combs have become linked and $\sigma_{tot}(A, \xi)$ becomes of the order of the nucleon-nucleon cross section. Finally, at $\alpha^1\xi \gg R^2$ there is only <u>one</u> slow comb at the fast nucleus; in this case the nucleus does not differ in principle in <u>any</u> way, from the point of view of the target, from a fast nucleon and therefore [3] the total cross section $\sigma_{tot}(A, \xi) = \sigma$, and the multiplicity is

$$n(A, \xi) = a(\xi - \Delta) + a\Delta \ln \Delta + a A \ln R\mu, \quad \Delta = \gamma_1 \frac{R^2}{2\alpha'}, \quad \gamma_1 \sim 1, \quad a = \bar{n}/\xi, \tag{6}$$

where the last two terms correspond to the products of the "fragmentation" of the nucleus (without nucleons from the disintegration of the nucleus).

The behavior of $F(A, \xi, \eta, p_\perp)$ is shown schematically in Fig. 1 (in the coordinate system in which the nucleus is at rest) as a function of the rapidity $\eta = \ln(E/m)$ of the observed particle.

2. Let us describe briefly how the considered picture can be corroborated more accurately.[2)] In the reggeon-diagram scheme, the inclusive cross section is obtained from the sum of the contributions of all the diagrams for the $(3 \to 3)$ amplitude. The most important circumstance in our case is that the pomeron branch cuts make no contribution to $f(p)$ (see [5]). Therefore at small E the only contributions that are not small are those of the diagrams of Fig. 2, in which the pomeron disintegration vertex Γ_ℓ lies below the vertex $\psi(p_\perp)$ that describes the emission of the observable particle (the analogous corrections to g have already been taken into account). At small $\eta < \ln(Rm)$ and arbitrary ξ, only the diagram of Fig. 2 with $\ell = 1$ (without the pomeron loops) is the significant; this leads to expression (3a). At large η, when the contributions with $\ell > 1$ are also not small, F(Z) acquires an additional dependence on η (but not on ξ and $p_\perp$):

$$F(A, \xi, \eta, p_\perp) = \phi(\eta, A) f(\xi, \eta, p_\perp), \tag{7}$$

where $\phi(\eta) = A$ at $\eta \lesssim \ln(Rm)$, and $\phi(\eta) \sim R^2/2\alpha'\eta$ at $\ln(Rm) \ll 2\alpha'\eta \ll R^2$, so that Eq. (4) holds at $\eta \sim \xi$. Further, since the rapidity θ on the pomerons (see Fig. 2) does not exceed $R^2/2\alpha'$ (since the vertices Γ_ℓ begin to vanish [3]), the spectrum goes over at $\eta > (R^2/2\alpha')$ into the spectrum per particle $(\phi \to 1)$. We emphasize once more that the dependence of F(A) on η is universal — the plot of Fig. 1 does not depend on ξ.

We can analogously obtain the higher inclusive cross sections. For example, the analog of (3a) for the two-particle cross section at $\varepsilon_1, \varepsilon_2 < \ln(Rm)$ is

$$F_2(A, p_1, p_2) \approx \sigma_{in}^{-1}(A) f(p_1) f(p_2) \left[A + \frac{A(A-1)}{2} \frac{\sigma}{\sigma_{in}(A)} c \right], \quad c \sim 1-2 \tag{8}$$

It should be noted that the expressions obtained in this manner for F(A), $F_2(A)$,.... are valid for all nuclei, not only for large A.

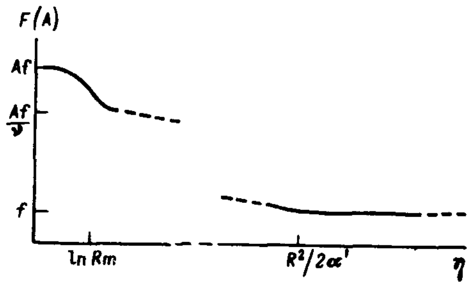

Fig. 1

3. Inelastic interactions of the nucleus $[A_1, R_1, EA_1]$ with the nucleus $[A_2, R_2, mA_2]$ can be considered analogously. At small $E \lesssim R_i m^2$ (when $\sigma_{in}(A_1, A_2) \simeq \pi b(R_1 + R_2)^2$) the diagrams with $\ell = 1$ are the essential ones for $[\varepsilon(\partial^3\sigma/\partial p^3)]_{A_1,A_2} \equiv F(A_1, A_2)$. Therefore

$$F(A_1, A_2; p) = A_1 A_2 f(p); \quad n(A_1 A_2 \xi) \approx \frac{A_1 A_2 \sigma}{\sigma_{in}(A_1, A_2)} \bar{n}. \tag{9}$$

When the energy is increased, a dip begins to appear in the central part of the spectrum, and the density of the produced particles will tend to the "single-comb" value. For arbitrary η and ξ we obtain

$$F(A_1, A_2; p) = \phi(\xi - \eta, A_1)\, \phi(\eta, A_2)\, f(p), \tag{10}$$

where $\phi(\eta, A)$ is the same function as in (7).

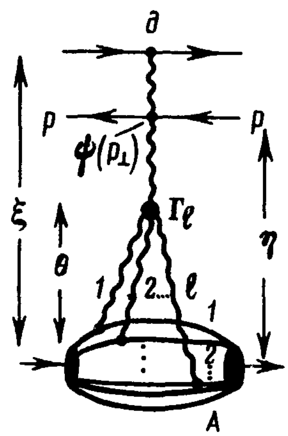

Fig. 2

4. In conclusion, we make two remarks. Assume that the spectra of the particles produced from the nuclei are indeed of the type proposed above. Then the detailed data on the behavior of these spectra at $\epsilon \gtrsim Rm\mu$ (in the laboratory frame of the nucleus) can yield information on the character of the linking of the parton combs [3]. We note also that the most important circumstance in our picture is the fact that the longitudinal distances, which play the important role in the hadron interactions, incrase like Em^{-2}; this is precisely why the usual cascades do not develop in the nucleus, and the fast part of the spectrum is not restructured.

The author thanks V. N. Gribov for numerous discussions and hints, and V. A. Abramovskii, E. V. Gedalin, I. D. Mandzhavidze, S. G. Matinyan, and K. A. Martirosyan for interest in the work.

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1)At small E, of course, the parton picture is quite arbitrary. In addition, the parton comb probably broadens in space near the slow end ($1/\mu$, and not $1/m$) owing to the interaction with the vacuum fluctuations. This can also lead to a certain "delay" and coalescence of the combs, and to a longitudinal stabilization of the slow partons of the nucuels at $E \sim 2Rm^2$, and not at $E \sim 2Rm\mu$.

2)A more detailed exposition will be published elsewhere.

[1] V. N. Gribov, Yad. Fiz. 9, 640 (1969) [Sov. J. Nuc. Phys. 9, 369 (1969)].
[2] R. P. Feynman, Photon-Hadron Interactions, W. A. Benjamin, 1972.
[3] V. N. Gribov, Yad. Fiz. 17, 603 (1973) [Sov. J. Nuc. Phys. 17, No. 3 (1973)].
[4] M. Gell-Mann and B. M. Udgaonkar, Phys. Rev. Lett. 8, 346 (1972).
[5] V. A. Abramovskii, V. N. Gribov, and O. V. Kancheli, Yad. Fiz. 18, 595 (1973) [Sov. J. Nuc. Phys. 18, No. 3 (1974)]; see also the Proceedings of the 16th Internat. Conf. on High Energy Physics, Vol. 1, p. 389.

GIANT SONDHEIMER OSCILLATIONS OF THE NERNST CONSTANT IN ELECTRON DRAGGING BY PHONONS

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Submitted 16 May 1973; resubmitted 31 August 1973
ZhETF Pis. Red. 18, No. 7, 469 — 473 (5 October 1973)

As shown by Sondheimer [1] some time ago, the conductivity of thin films in external magnetic fields oscillates as a function of the parameter $\beta = d/r_H$, where d is the characteristic transverse dimension of the sample and r_H is the radius of curvature of the electron trajectory in the magnetic field. It is natural to expect analogous oscillations to be experienced also by other kinetic coefficients, particularly the thermoelectric and thermomagnetic ones. The latter, however, are determined in semiconductors and semimetals at low temperatures mainly by the dragging of the electrons by phonons, thus distinguishing them from the picture considered by Sondheimer. It is shown in the present paper that even in the absence of dragging the Sondheimer oscillations of the Nernst constant can be so large that this constant can alternate in sign. The dragging effect, however, can increase the amplitude of these oscillations by several orders of magnitude.

We consider an isotropic sample in the form of a plane-parallel plate of thickness d, bounded in the z direction and infinite along the axes x and y. The magnetic field is directed along the z axis and the temperature gradient along x. The kinetic equations for the electrons and phonons in the presence of a magnetic field and a temperature gradient are

$$\frac{\partial f_p^1}{\partial z} + \frac{e}{v_z}\left\{E + \frac{1}{c}[\mathbf{v}\times\mathbf{H}]\right\}\nabla_p f_p + \frac{\left(\mathbf{v}\nabla_r f_p^0\right)}{v_z} = -\frac{f_p^1}{\tau_p v_z} + \hat{D}^1\{f_p^0, F_q^1\}, \quad (1)$$

$$s\frac{\partial F_q^1}{\partial z} s\nabla T\frac{\partial F_q^0(\omega)}{\partial T} = \hat{I}\{F_q\}, \quad (2)$$

where

$$f_p = f_p^0 + f_p^1, \quad f_p^1 = (v_x\Psi_x + v_y\Psi_y), \quad F_q = F_q^0 + F_q^1.$$