

According to (5), in a lossless medium the quantity  $J$  tends with increasing  $z$  to the maximum limit  $|A_{S0}|^2 + (\omega_S/\omega_P)I_0$  permitted by the Manley Rowe relation, meaning that all the optical noise power can be transferred to the monochromatic component of scattered light whose frequency coincides with the priming frequency. The rate of this transfer is determined in this case by the quantity  $\mu = 1/\Delta\omega T_2$  (see the figure). This result is explained by the picture of transformation of the spectra, which can be obtained from the fluctuation equations (2) and (4) ( $A_p = A_p, Q = Q$ ), analogous to (b)

$$G_p(\omega, z) = G(\omega) \exp \left\{ - \frac{\omega_p}{\omega_S} g F(\omega) \int_0^z J(z') dz' \right\},$$

$$G_Q(\omega, z) = \sigma_Q^2 T_2^2 F(\omega) G_p(\omega, z),$$

namely, the first to be transferred is the energy contained at the center of the spectral line of the noise, followed by the energy in the wings of the spectrum, owing to the beats of the correlated fluctuations of the pump and of the phonons. As seen from the last formula, the relative role of the wings of the phonon-wave spectrum increases in the energy transfer with increasing  $z$ .

5. The Dyson-equations method was used by us also to analyze the propagation of a noise-filled light pulse in a two-level system, with saturation taken into account. These results will be published in another article.

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#### ELECTRODYNAMICS OF SURFACE SUPERCONDUCTIVITY

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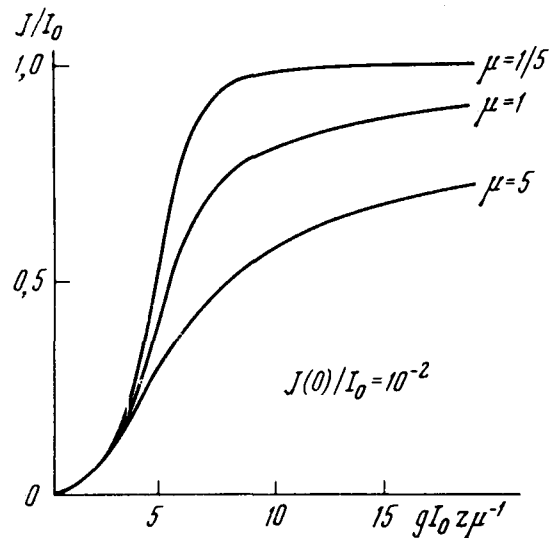
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The energy spectrum of the excitations and the surface of impedance of a metal are investigated under conditions when surface superconductivity is present.

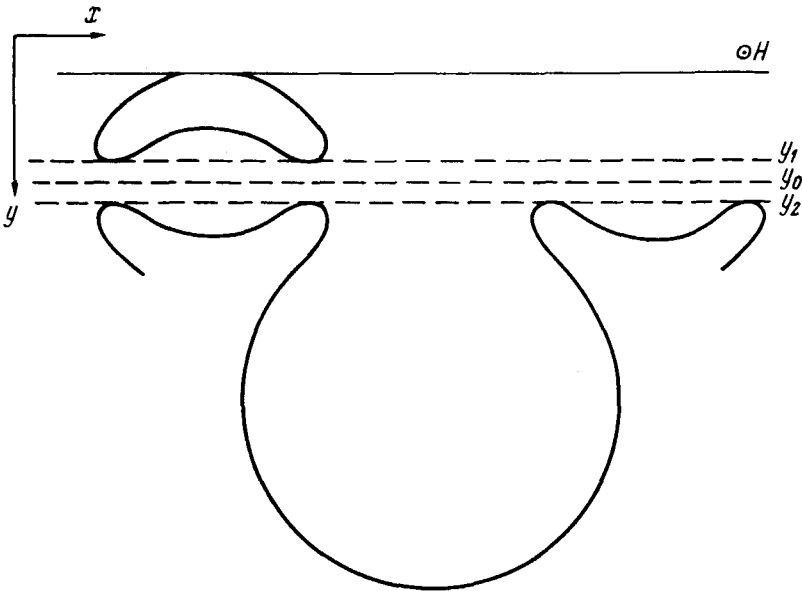
We consider the energy spectrum of the excitations of a semi-infinite superconductor in an



external magnetic field  $H$  parallel to the surface ( $H_{C2}, H_C \leq H \leq H_{C3}$ , where  $H_C, H_{C2}$ , and  $H_{C3}$  are respectively the critical and second and third critical fields). It is known [1, 2] that under these conditions the gap  $\Delta$  is different from zero only near the surface, at a distance on the order of  $\xi$  ( $\xi$  is the characteristic scale of variation of  $\Delta$ ). If we choose the coordinate axes as in Fig. 1, then the magnetic field and  $|\Delta|$  depend only on the  $y$  coordinate. We choose also a gauge in which the gap  $\Delta$  is real, and the vector potential  $\vec{A} = (A_x, 0, 0)$  also depends only on  $y$ ; this is possible because the phase of  $\Delta$  is a function of  $x$  and  $z$ . The problem of determining the energy spectrum and the eigenfunctions of the excitations (the eigenfunctions of the Gor'kov equations [3]) reduces thus to a one-dimensional problem, and it can be solved by using a quasi-classical approximation [4, 5].

Near  $H_{C3}$ , in the region of applicability of the Ginzburg-Landau equations, the magnetic field can be regarded as homogeneous, and we obtain for  $A_x$  the expression

$$A_x = -H_y + \text{const} = -H(y - y_0), \quad y_0 = 0,59 \xi \frac{H_C}{H}. \quad (1)$$



We note that the vector potential vanishes at the point  $y_0$  (approximately the midpoint of the layer); this is natural, since the current is determined in the chosen gauge by the London expression and can be of alternating sign, since the total current is equal to zero. This property of the vector potential with our choice of gauge remains in force in the entire region of surface conductivity; all that changes is the expression for  $y_0$ .

The energy spectrum of the excitation is determined in the quasiclassical approximation from the Bohr rules

$$\oint p_y(y) dy = 2\pi n \hbar \quad (2)$$

where  $p_y$  is the classical momentum of the excitation along the  $y$  axis, and  $n$  is an integer. The criterion for the applicability of (2), as usual, is  $n \gg 1$  [6], and is automatically confirmed. To determine the  $p_y(y)$  dependence we can use the Bogolyubov-deGennes equations [1], where it is necessary, unlike in [4, 5], to retain also the terms quadratic in the magnetic field, since the magnetic field does not attenuate in the interior of the metal. It is easily found that

$$p_y = \pm \sqrt{2m \left\{ \mu_1 - \frac{1}{2m} \left( \frac{eA_x}{c} \right)^2 \pm \sqrt{(\epsilon - \rho)^2 - \Delta^2} \right\}},$$

$$\mu_1 = \epsilon_F - \frac{1}{2m} (P_x^2 + P_z^2), \quad \rho = \frac{e}{mc} P_x A_x. \quad (3)$$

Here  $\epsilon_F$  is the Fermi energy,  $P_x$  and  $P_z$  are the conserved (by virtue of the homogeneity)

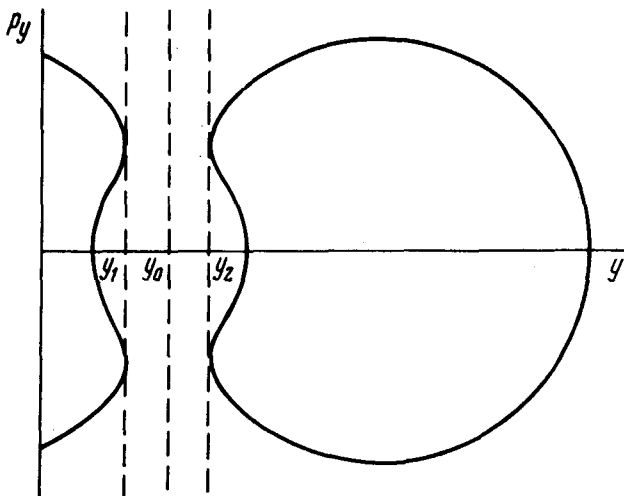


Fig. 2

momenta of the excitations along the x and z axis, and  $\epsilon$  is the energy of the excitations. The characteristic trajectories in the coordinate and phase spaces are shown in Figs. 1 and 2, respectively. It is seen from (2) and (3) that there are always excitations with  $\epsilon = 0$ , since

$$\rho(0) \geq \Delta(0) \quad (P_x \sim p_F).$$

Some of the excitations (the surface ones) cannot penetrate into the metal, since there is a turning point  $y_1$  ( $y_1 < y_0$ ) where  $\rho(y_1) = \Delta(y_1)$ . They move with a frequency

$$\Omega \sqrt{R/\xi} \gg \Omega$$

( $R$  and  $\Omega$  are the cyclotron radius and frequency, respectively), and mainly in analogy with the motion considered in [4, 5]. Nor can excitations moving from the interior of the metal (volume excitations) reach the surface  $y = y_0$  at  $\epsilon = 0$ , in view of the existence of the turning point  $y_2$  ( $y_2 > y_0$ ) at which  $|\rho(y_2)| = \Delta(y_2)$ . The frequency of the corresponding motion differs slightly (by  $\sqrt{(\xi/R)\Omega} \ll \Omega$ ) from the cyclotron frequency. We emphasize that the presence of the turning points  $y_1$  and  $y_2$  is due to the vanishing of the vector potential at the point  $y_0$ .

Let us see now what occurs when an electromagnetic wave is incident on the metal. At  $T < T_C$ , the region considered from now on, the energy absorption occurs mainly in the regions where there are excitations with  $\epsilon = 0$ , i.e., in the surface layer ( $y < y_1$ ) and in the volume ( $y > y_2$ ). The dependence of the impedance on the external magnetic field and on the frequency is determined essentially by the relation between the other parameters of the problem and the depth of penetration of the alternating field  $\delta$ . Deferring the detailed exposition to another article, we stop to discuss here the case when  $\delta > \xi$ , a case that can be realized if the frequency is not too low or if the magnetic field is sufficiently close to  $H_{C3}$ . In this approximation, the surface impedance  $Z$  can obviously be expressed in the form

$$\frac{1}{Z} = \frac{1}{Z_{\text{sur}}} + \frac{1}{Z_{\text{vol}}},$$

where  $Z_{\text{sur}}$  is determined by the nondissipative superconducting and dissipative surface currents, and  $Z_{\text{vol}}$  is determined by the current of the volume excitations. The expression for  $Z_{\text{vol}}$  is analogous, with a relative accuracy on the order of  $\sqrt{\xi/R} \ll 1$ , to the corresponding expression for the impedance of a normal metal with strictly specular reflection from the surface. In particular, there should also be observed cyclotron resonance at a slightly modified (to the extent that  $\sqrt{\xi/R}$  is small) cyclotron frequency. The value of  $Z_{\text{sur}}$  is determined by the effective conductivity of the surface layer, and if the scattering by the surface is diffuse, then the effective free-path time is of the order of  $\sqrt{Ry_0}/v_F$ , and

$$\sigma_{\text{eff}} \sim -i \frac{ne^2}{m\omega} + \sigma_0 \frac{\sqrt{Ry_0}}{\ell} \frac{y_0}{\xi}, \quad Z_{\text{sur}} \sim \frac{1}{\sigma_{\text{eff}} \xi}.$$

We see that at frequencies that are not too low, or in a field sufficiently close to  $H_{C3}$ , a situation is possible wherein

$$Z_{\text{sur}} \sim \frac{\ell}{\sqrt{Ry_0} y_0 \sigma_0} \propto H^2.$$

Let us stop also to discuss the obvious anisotropy of the impedance  $Z_{\text{sur}}$ , and hence of the entire surface impedance  $Z$ , with respect to the angle between the constant and alternating magnetic fields (currents). In our approximation  $\delta > \xi$ , when  $H_1 \perp H$  ( $H_1$  is the amplitude of the alternating magnetic field), the gap  $\Delta$  does not change in the linear approximation in  $H_1$ , but at  $H_1 \parallel H$  the change of  $\Delta$  can obviously be expressed in the form

$$\Delta_1 = \frac{\partial \Delta}{\partial H} H_1.$$

The effect of the change in the gap  $\Delta$  can be easily estimated from the excitation dispersion law (2). The relative contribution made to the absorption by the change of the gap is of the order of

$$\frac{\xi}{\delta} \frac{H_{c3}}{\Delta} \frac{\partial \Delta}{\partial H}$$

and can be appreciable, in spite of the small factor  $\xi/\delta$  in this approximation, because  $\partial\Delta/\partial H$  is large. Such an anisotropy was observed experimentally in [7].

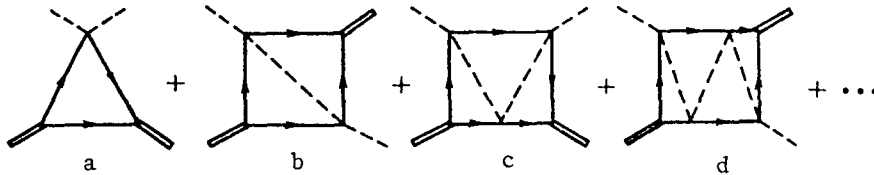
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#### PION-DEUTERON AND PION-NUCLEON SCATTERING LENGTHS

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1. Pion-deuteron scattering at low energies is of interest from two points of view. First, this is the simplest process of pion interaction with nuclei, and its use as an example makes it easiest to verify the accuracy of the assumptions customarily made upon introduction of the Kisslinger-Ericson potential or in other theoretical schemes. Second, one can hope that measurement of the pion-deuteron scattering length can yield additional information on the parameter  $b_0$ , which is the average of the  $\pi^-n$  and  $\pi^-p$  scattering lengths and is presently known with very poor accuracy, although it is of interest in a number of theories (e.g., in the PCAC hypothesis its value is rigorously equal to zero).



The  $\pi d$ -scattering amplitude was obtained in [1] by summing a series of diagrams (see the figure), and the expression used for the  $\pi N$  scattering was

$$f_{\pi N} = b_0 + b_1 \vec{t} \cdot \vec{\tau}, \quad (1)$$

where  $\vec{t}$  and  $\vec{\tau}$  are the pion and nucleon isospin operators. The main contribution is made by single- and multiple-scattering terms, in which the p-wave part of the  $\pi N$  interaction was also taken into account, and in the double-scattering term allowance was made also for effects connected with the kinetic energy of the nucleons in the intermediate state. The numerical values of the parameters  $b_0$  and  $b_1$  were taken from the review [2]:  $b_0 = -0.017 \pm 0.006 F$  and  $b_1 = -0.097 \pm 0.007 F$ . As shown in [3], the contribution of the diagrams with virtual re-scattering of the nucleons can be neglected.

A group at the University of London has recently performed an exact measurement of the energy of the  $2p - 1s$  transition in a pionic deuterium atom, and obtained a preliminary result for the  $\pi d$ -scattering length