GAMMA-QUANTUM EMISSION BY NUCLEI EXCITED BY HIGH-ENERGY HADRONS

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It is shown with the aid of Glauber's theorem that the angular distribution of the gamma radiation is determined mainly by [J/2] + 1 parameters. The remaining parameters due to the particle spins are small quantities.

The emission of gamma quanta by nuclei colliding with high-energy proton was investigated in a recent experiment [1]. In the present paper we obtain the bounds on the angular distribution of the emission of nuclei having a 0^+ ground state. The excited states are assumed to have natural parity. We confine ourselves for simplicity to the scattering of pions by nuclei, but the obtained estimates hold also for other hadrons.

Let the ground and excited states of the nucleus be described by the wave functions Ψ_0 and Ψ_f, respectively, and let the state f have a spin J ($\hbar = 1$) whose projection on the Z axis is denoted M, and let its parity P be $(-1)^J$. The amplitude of the transition of the nucleus from the ground state to the state f is, according to [2],

$$F_{f,0}(q) = \frac{ik}{2\pi}\int e^{iqb}(\Psi_f, \Gamma\,\Psi_0)\,d^2b\,. \tag{1}$$

Here $k = |\vec{k}|$, $\vec{k}$ is the incident-meson momentum directed along the Z axis, $\vec{b}$ is the impact vector, $\vec{q}$ is the momentum transferred to the nucleus, and the "profile" function Γ is given by

$$\Gamma = \sum_{n=1}^{A}\frac{(-1)^{n+1}(2\pi ik)^{-n}}{n!\,(A-n)!}\,S\int\prod_{\ell=1}^{n}[f_\ell e^{-i\Delta_\ell(b-s_\ell)}]\prod_{\beta=1}^{n}d^2\Delta_\beta\,. \tag{2}$$

In (2), S is the sum of all the permutation operators of A particles of the target, $\vec{s}_j$ is the projection of the radius vector of the j-th nucleon of the nucleus on the XY plane, and f_j is the amplitude for the elastic scattering of a pion by the j-th particle [3]:

$$f_j = A_j(E,\Delta_j) + B_j(E,\Delta_j)(\vec{\sigma}_j\,\vec{\nu}_j). \tag{3}$$

The Pauli matrices $\vec{\sigma}_j$ in (3) act on the spin variables of the nucleons, $\vec{\nu}_j$ is a unit vector in the direction of $\vec{k}_j \times \vec{k}_{j+1}$, and $\vec{k}_j$ is the momentum of the meson prior to collision with the j-th particle. The scalar functions A_j and B_j depend on the meson energy E and on Δ_j ($\vec{\Delta}_j = \vec{k}_j - \vec{k}_{j+1}$). As $\Delta_j \to 0$ we have

$$B_j = C_j(E)\,\Delta_j/\mu + \ldots, \tag{4}$$

where μ is a certain characteristic mass ($\mu \sim 1$ GeV).

We define an operator U_z that acts only on the spatial and spin variables of the nucleons of the nucleus, by the relation

$$U_z = U_p U_3. \tag{5}$$

In (5), U_p is the inversion operator and U_3 is the operator of rotation through 180° about the Z axis. Using U_z, we write

$$F_{f,0} = \frac{ik}{2\pi}\int e^{iqb}(\Psi_f,\ U_z^2\Gamma U_z^2\,\Psi_0)\,d^2b = \\ = (-1)^M P\frac{ik}{2\pi}\int e^{iqb}(\Psi_f,\ U_z\Gamma U_z\Psi_0)\,d^2b. \tag{6}$$

We denote by M_+ and M_- the projections M satisfying relations (7) and (8), respectively; here

$$(-1)^M P = 1, \tag{7}$$

$$(-1)^M P = -1. \tag{8}$$

If we now define Γ_+ and Γ_- by the formulas

$$\Gamma_\pm = \frac{1}{2}(\Gamma \pm U_z \Gamma U_z), \tag{9}$$

then only the "profile"function $\Gamma_+(\Gamma_-)$ will contribute to the amplitudes $F_{fM_+,0}$ ($F_{fM_-,0}$). The important values in Glauber scattering are $q \lesssim R^{-1}$, where R is a distance on the order of the nuclear radius, and all $\Delta_j \lesssim R^{-1}$. Retaining in (2) only the terms of zeroth and first order in $(\mu R)^{-1}$, we obtain from (2), (3), and (4)

$$\Gamma_+ = \sum_{n=1}^{A} \frac{(-1)^{n+1}(2\pi ik)^{-n}}{n!(A-n)!} S \int \prod_{\ell=1}^{n} \left[A_\ell e^{-i\vec{\Delta}_\ell (b - s_\ell)}\right] \prod_{\beta=1}^{n} d^2\Delta_\beta, \tag{10}$$

$$\Gamma_- = \sum_{n=1}^{A} \frac{(-1)^{n+1}(2\pi ik)^{-n}}{n!(A-n)!} S \int \sum_{i=1}^{n} \Big[C_i(\Delta_i/\mu)(\vec{\sigma}_i \vec{\nu}_i) \times$$
$$\times e^{-i\vec{\Delta}_i (b - s_i)}\Big] \prod_{\substack{\ell=1,\\ \ell \neq i}}^{n} [A_\ell e^{-i\vec{\Delta}_\ell (b - s_\ell)}] \prod_{\beta=1}^{n} d^2\Delta_\beta. \tag{11}$$

From (1), (10), and (11) we conclude that

$$|F_{fM_-,0}(q)/F_{fM_+,0}(q)|^2 \lesssim (\mu R)^{-2} |C_i(E)/A_i(E,0)|^2. \tag{12}$$

Since $C_j/A_j \lesssim 1$ (and decreases with increasing energy), and $(\mu R)^2 \gg 1$, the states predominantly excited in Glauber scattering are those with projections M_+. This is manifest in the gamma emission of the nuclei.

We consider only the radiative transition $f \to 0^+$. The formulas for the more general case can be found, e.g., in [4]. The density matrix of the excited state of the nucleus is diagonal after averaging over the momentum transfers. We denote its diagonal elements by ξ_M; the angular distribution of the radiation is then described by the formula

$$W(\mathbf{n}) = \sum_{M_+} \xi_{M_+} |\mathbf{Y}^{(1)}_{JM_+}(\mathbf{n})|^2 + \sum_{M_-} \xi_{M_-} |\mathbf{Y}^{(1)}_{JM_-}(\mathbf{n})|^2. \tag{13}$$

In (13), $\vec{n}$ is a unit vector in the photon-observation direction, and $Y^{(1)}_{JM}$ is a spherical vector of the electric type. The expression in the right-hand side of (13) is a polynomial of degree J in $\cos\theta$ ($\cos\theta = n_z$). The number of independent parameters ξ_M is J + 1, by virtue of the relation $\xi_M = \xi_{-M}$, so that all the parameters can be uniquely determined from experiment. Since

$$\xi_{M_-}/\xi_{M_+} = \int |F_{fM_-,0}|^2 d^2q / \int |F_{fM_+,0}|^2 d^2q, \tag{14}$$

we get from (12) and (14) the sought estimate

$$\xi_{M_-}/\xi_{M_+} \lesssim (\mu R)^{-2} |C_i(E)/A_i(E,0)|^2. \tag{15}$$

We conclude from (15) that the contribution of the first sum predominates in (13). To find, for example, the total cross section for the excitation of the level f, as was done in [1], it is therefore necessary to determine only the quantities ξ_{M_+}. At μ = 1 GeV, R = 2 F, and $C_j/A_j = 1$, the right-hand side of (15) equals 10^{-2}, and if experiment shows that $\xi_{M_-}/\xi_{M_+} \gg 10^{-2}$, this will be an indication of the importance of spin effects in Glauber scattering. We did not take into account the influence of the nuclear motion on the angular distribution of the radiation, and have neglected the longitudinal part of the momentum transfer. Simple estimates show, however, that at $E_f - E_0 \lesssim 10$ MeV these factors do not lead to violation of (15).

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