Anisotropy of the acoustoelectric effect in metals

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A formula is derived for the acoustoelectric current (and voltage) in a metal at an arbitrary dispersion of the conduction electrons; this formula accounts in principle for the experimental results.

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One of us, continuing the investigations of Ref. 1, observed an anisotropic acoustoelectric voltage (Fig. 1) in a single-crystal tin sample. Although there is no doubt that the acoustoelectric effect is due to the same interaction between the sound flux and the conduction electrons which leads to the damping of the sound, the two phenomena have different anisotropies (see Fig. 1). Our purpose in the present communication is to derive a formula that describes the acoustoelectric effect, without simplifying assumptions concerning the dispersion of the metal electrons,[1)] and to explain the nature of the difference between the angular dependences of the acoustoelectric current $J(\mathbf{n})$ and the sound absorption coefficient $\Gamma(\mathbf{n})(\mathbf{q}=q\mathbf{n}$ is the wave vector of the sound wave; in the experiment $ql\approx 20$, so that it can be assumed that $ql\gg 1$, where l is the electron mean free path). We describe the acoustic flux by a δ-function distribution in $\mathbf{k}$-space

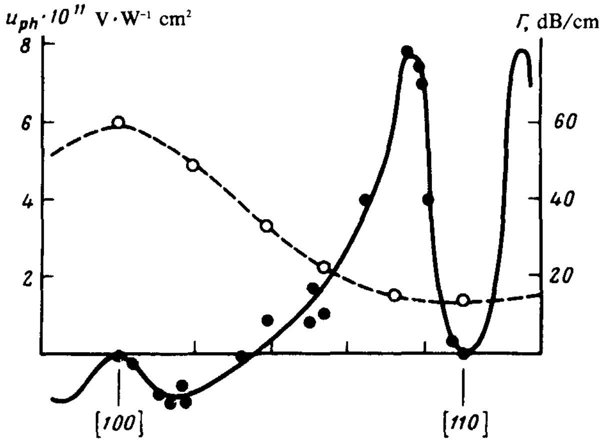

FIG. 1. Anistoropy of the acoustoelectric voltage $v_{\rm ph}$ (solid curve) and absorption of ultrasound Γ (dashed) in accord with the data of Ref. 2 in the [001] plane for tin and longitudinal sound oscillations.

$$N(\mathbf{k}) = \frac{W}{\hbar s\,\omega_{\mathbf{k}}}\,\delta(\mathbf{k}-\hbar\mathbf{q}), \qquad (1)$$

where W is the flux density of sound with frequency $\omega_{\mathbf{k}}$, and s is the speed of sound. We start out in the analysis with the kinetic equations for the electron $f(\mathbf{p})$ and phonon $N(\mathbf{k})$ distribution functions:

$$\left(\frac{\partial f}{\partial t}\right)_{\text{stat}} = \left(\frac{\partial f}{\partial t}\right)_{st} + \frac{\pi}{\hbar^2 \rho} \sum_{\mathbf{k}} \frac{|\Lambda|^2 k^2 N(\mathbf{k})}{\omega_{\mathbf{k}}} \{ (f(\mathbf{p}+\mathbf{k}) - f(\mathbf{p}))\,\delta(\epsilon(\mathbf{p}+\mathbf{k}) \tag{2}$$

$$- \epsilon(\mathbf{p}) - \hbar\omega_{\mathbf{k}}) + (f(\mathbf{p}-\mathbf{k}) - f(\mathbf{p}))\,\delta(\epsilon(\mathbf{p}-\mathbf{k}) - \epsilon(\mathbf{p}) + \hbar\omega_{\mathbf{k}}) \} = 0,$$

$$\frac{\partial N(\mathbf{k})}{\partial t} = \frac{2\pi k^2}{\hbar^2 \rho \omega_{\mathbf{k}}} N(\mathbf{k}) \sum_{\mathbf{p}} |\Lambda|^2 (f(\mathbf{p}+\mathbf{k}) - f(\mathbf{p}))\,\delta(\epsilon(\mathbf{p}+\mathbf{k}) - \epsilon(\mathbf{p}) - \hbar\omega_{\mathbf{k}}), \tag{3}$$

Here ρ is the density of the metal and Λ is the corresponding component of the deformation potential. The collision integral $(\partial f/\partial t)_{st}$ includes the interaction of the electrons with the thermal phonons, with the impurities, and with one another.

From (3) we obtain the well-known expression for the damping coefficient of the sound flux[4]

$$\Gamma(\mathbf{n}) = \frac{2\pi\omega_{\mathbf{q}}}{(2\pi\hbar)^3 \rho s^2} \int \frac{|\Lambda|^2}{v_F^2}\, \delta\left(\vec{\nu}\mathbf{n} - \frac{s}{v_F}\right) dS_F\,, \tag{4}$$

$$\vec{\nu} = \mathbf{v}/v\,,$$

and we linearize Eq. (2) with respect to W, by replacing $f(\mathbf{p})$ with $f_F(\mathbf{p}) + f_1$, where $f_F(\mathbf{p})$ is the equilibrium Fermi function and $f_1 \sim W$. Noting that because of (1) the right-hand side of (2) contains δ functions, we verify that we can neglect in the collision integral $(\partial f/\partial t)_{st}$ the "arrival" terms, i.e.,

$$\left(\frac{\partial f}{\partial t}\right)_{st} = -\frac{f_1}{\tau_{\mathbf{p}}}\,, \text{ where } \tau_{\mathbf{p}}^{-1} = \int w_{\mathbf{pp}'}\, d\mathbf{p}'\,,$$

and w'_{pp} is the suitably normalized probability of the transition from the state $|\mathbf{p}\rangle$ to the state $|\mathbf{p}'\rangle$ via all the scattering mechanisms. Thus,

$$f_1 = \frac{\pi\tau_{\mathbf{p}} |\Lambda|^2 W}{\hbar \rho s^3} \{ [f_F(\mathbf{p}+\hbar\mathbf{q}) - f_F(\mathbf{p})]\,\delta(\epsilon(\mathbf{p}+\hbar\mathbf{q}) - \epsilon(\mathbf{p}) - \hbar\omega_{\mathbf{q}})$$

$$+ [f_F(\mathbf{p}-\hbar\mathbf{q}) - f_F(\mathbf{p})]\,\delta(\epsilon(\mathbf{p}-\hbar\mathbf{q}) - \epsilon(\mathbf{p}) + \hbar\omega_{\mathbf{q}}) \}\,. \tag{5}$$

The acoustoelectric current $J(\mathbf{n})$ in the direction of $\mathbf{n}=\mathbf{q}/q$ is expressed by the formula

$$J(\mathbf{n}) = \frac{2\pi e W}{(2\pi\hbar)^3 \rho s^3} \left\{ \int \tau_{\mathbf{p}} |\Lambda|^2 \frac{\partial f_F}{\partial \epsilon} \hbar\omega_{\mathbf{q}} \mathbf{v}\mathbf{n}\, \delta(\epsilon(\mathbf{p}+\hbar\mathbf{q}) - \epsilon(\mathbf{p}) - \hbar\omega_{\mathbf{q}})\, d\mathbf{p} \right.$$

$$\left. - \int \tau_p |\Lambda|^2 \frac{\partial f_F}{\partial \epsilon} \hbar\omega_{\mathbf{q}} \mathbf{v}\mathbf{n}\, \delta(\epsilon(\mathbf{p}-\hbar\mathbf{q}) - \epsilon(\mathbf{p}) + \hbar\omega_{\mathbf{q}})\, d\mathbf{p} \right\}. \qquad (6)$$

Retaining in the arguments of the δ functions the terms proportional to q^2, we have:

$$J(\mathbf{n}) = \frac{2\pi e W \omega_{\mathbf{q}}}{(2\pi\hbar)^3 \rho s^3} \int \frac{l_{\mathbf{p}} |\Lambda|^2}{v_F^3} \left[\frac{\partial^2 \epsilon}{\partial p_n^2} \right]_F \delta\left(\vec{v}\mathbf{n} - \frac{s}{v_F}\right) dS_F, \qquad (7)$$

where $l_{\mathbf{p}} = \tau_{\mathbf{p}} v_F$ is the "departure" electron mean free path.

Comparison of formulas (7) and (4) shows the following: 1) under the simplest assumptions concerning the dispersion law $[\epsilon(\mathbf{p})=p^2/2m]$ we obtain the Weinreich relation[3] $E=\Gamma W/ens$; 2) the acoustoelectric effect, in contrast to sound absorption (even at $ql \gg 1$) depends substantially on the dissipation mechanisms; 3) in both cases, electrons on the "belts" participate; 4) owing to the reciprocal-effective-mass tensor $[\partial^2\epsilon/\partial p_n^2]_F$, the anisotropy of $J(\mathbf{n})$ can greatly differ from the anisotropy of $\Gamma(\mathbf{n})$.

We see that the sign of the effect is determined by the sign of the reciprocal effective-mass tensor $[\partial^2\epsilon/\partial p_n^2]_F$ along the $\mathbf{n}$ direction. It must be emphasized that a negative contribution to $J(\mathbf{n})/e$ can be made by the electron Fermi surface, and a positive contribution by the hole surface. Figure 2 shows an electron surface of the

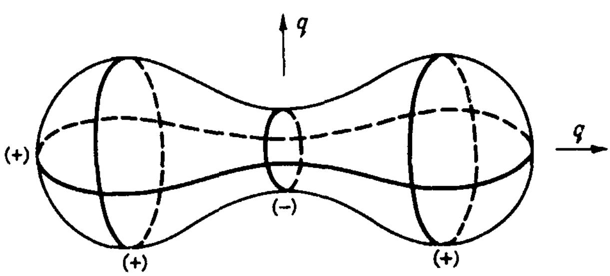

FIG. 2. Topology of the "belts" on a Fermi surface of a "dumbbell" type. The sign of the contribution of each "belt" to $J(\mathbf{n})/e$ is marked.

"dumbbell" type.

At $\mathbf{q}=q\mathbf{n}$ parallel to the "dumbbell" axis, the contribution from the central "belt" is negative, and at q perpendicular to the axis the contribution from the existing is positive.[2)] The change in the direction of propagation of the sound should lead to singularities in the angular dependence of $J(\mathbf{n})$, due to the change of the topology of the "belt" (see Ref. 5).

The acoustoelectric field is $E(\mathbf{n}) = \rho_n J(\mathbf{n})$, where ρ_n is the resistivity along $\mathbf{n}$. The anisotropy of $E(\mathbf{n})$ should not differ substantially from the anisotropy of $J(\mathbf{n})$, since the resistivity ρ_n is determined by all the Fermi electrons and depends only on the symmetry class of the crystal.

A detailed comparison of the theory with experiment calls for numerical calculations that make use of a definite model of the Fermi surface.

[1)] The theory of the acoustoelectric effect is the subject of relatively many papers (see, e.g., Ref. 3 and the literature cited therein). We, however, know of no analysis of the difference between the anisotropies of sound damping and of the acoustoelectric effect in metals.

[2)] We note that the contribution made to $J(\mathbf{n})$ by a "belt" of small dimension can be anomalously large (cf. Ref. 6). Reversal of the sign of the effect is therefore perfectly feasible even in the simplest case shown in Fig. 2.

[1] N.V. Zavaritskiĭ, Pis'ma Zh. Eksp. Teor. Fiz. **25**, 61 (1977) [JETP Lett. **25**, 55 (1977)].

[2] Y.M. Perz and E.R. Dobbs, Proc. R. Soc. London **297**, 408 (1967).

[3] V.L. Gurevich, Fiz. Tekh. Poluprovodn. **2**, 1557 (1968) [Sov. Phys. Semicond. **2**, 1299 (1968)]; N. Mikoshiba, J. Appl. Phys. **34**, 510 (1963); S.G. Eckshtein, J. Appl. Phys. **35**, 2702 (1964).

[4] A.I. Akhiezer, M.I. Kaganov, and G.Ya. Lyubarskiĭ, Zh. Eksp. Teor. Fiz. **32**, 837 (1957) [Sov. Phys. JETP. **5**, 685 (1957)].

[5] G.T. Avanesyan, M.I. Kaganov, and T.Yu. Lisovskaya, Pis'ma Zh. Eksp. Teor. Fiz. **25**, 381 (1977) [JETP Lett. **25**, 355 (1977)].

[6] V.N. Davydov and M.I. Kaganov, Pis'ma Zh. Eksp. Teor. Fiz. **16**, 133 (1972) [JETP Lett. **16**, 92 (1972)]; Zh. Eksp. Teor. Fiz. **67**, 1491 (1974) [Sov. Phys. JETP **40**, 741 (1975)].