

Spin waves in pulsed NMR experiments in the B phase of He^3

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The dispersion law of spin waves in the B phase of He^3 in a strong magnetic field is determined for arbitrary values of the initial angle between the magnetization and the field. Instability of the homogeneous precession is observed for a large class of initial conditions.

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In pulsed NMR experiments one studies the dependence of the frequency of the spatially homogeneous precession of the magnetization \mathbf{S} in an external magnetic field \mathbf{H}_0 on the initial angle β between \mathbf{S} and \mathbf{H}_0 . These experiments have already yielded substantial information on the properties of the superfluid phases of He^3 (Ref. 1). It is of interest to consider the behavior of small inhomogeneous perturbations of the homogeneous precession, i.e., spin waves, and by the same token investigate the stability of the pattern of homogeneous precession. In this article we present results of such an investigation for the superfluid B phase of He^3 , both for different values of β and for different orientation of \mathbf{S} relative to the vector \mathbf{n} that characterizes the anisotropy of the order parameter in the B phase. The external field \mathbf{H}_0 is assumed to be strong in the sense that the corresponding Larmor frequency ω_L is much larger than the frequency Ω of the longitudinal oscillations of the magnetization.

We confine ourselves to low-frequency spin waves, i.e., frequencies (or deviation of the frequency from the Larmor value) ω much less than ω_L , and are interested only in the asymptotic solution of the equations of motion in the principal order in the small parameter $(\omega/\omega_L)^2$. The procedure of finding the small corrections to the Larmor precession for the superfluid phases of He^3 is described in detail in Ref. 2. The case considered here differs in that besides the corrections due to the spin—orbit interaction it is necessary to take into account also the corrections due to the spatial inhomogeneity of the order parameter. To this end it is necessary to add to the Hamiltonian (6) of Ref. 2 terms that depend on the gradients of the Euler angles α, β and γ , which describe the rotation of the order parameter away from its initial state [see formula (19) of Ref. 3], and then average over the “fast” variables α and γ and express the result in terms of the canonical variables (Φ, S) and (α, P) . As a result, the Hamiltonian that describes the He^3 - B spin system takes the form

$$\begin{aligned} \overline{\mathcal{H}} = & \frac{(S-1)^2}{2} - P + V(\Phi, P) + \frac{c^2}{2} \left[\Phi_1^2 + \Phi_2^2 - \frac{P(6-P)}{4} (\alpha_1^2 + \alpha_2^2) \right] \\ & + 2P(\alpha_1\Phi_1 + \alpha_2\Phi_2) - \frac{3}{4P(P+2)} (P_1^2 + P_2^2) \end{aligned}$$

$$\left. + \frac{1}{2} \Phi_3^2 - \frac{P(P+4)}{2} \alpha_3^2 + P \alpha_3 \Phi_3 - \frac{1}{P(P+2)} P_3^2 \right] \quad (1)$$

Subscripts denote differentiation with respect to a coordinate, and c is a constant of dimension of velocity, which will henceforth be set equal to unity. The remaining symbols are the same as in Ref. 2.

Using the standard formalism⁴ we can write down the equations of motion that correspond to the Hamiltonian (1). In the spatially homogeneous case they reduce to the system (8), (9) of Ref. 2 and, as already shown, have a solution that describes magnetization precession with frequency $\dot{a} = -1 + \partial V / \partial P$, where $P = P_0 = \text{const}$, $\Phi = \Phi_0 = \text{const}$, and Φ_0 is the root of the equation $\partial V(\Phi, P_0) / \partial \Phi = 0$. Linearization of the equations of motion about this solution leads to a system of equations that describe the behavior of small inhomogeneous perturbations. The system obtained in this manner has solutions $\propto \exp[i(\mathbf{k} \cdot \mathbf{r} - \omega t)]$ corresponding to spin waves. The dispersion law of these waves is anisotropic and we consider here the dispersion equation corresponding to waves with $\mathbf{k} \parallel \hat{z}$, i.e., propagating along the magnetic-field direction:

$$\left(\omega^2 - V_{\Phi\Phi} - \frac{k^2}{2} \right) \omega^2 = \frac{1}{4} P k^2 \left[V_{PP} - \frac{k^2}{P(P+2)} \right] \times [P k^2 + (P+4)(2V_{\Phi\Phi} + k^2 - 2\omega^2)] - P(P+4) \frac{k^2}{2} V_{P\Phi} \quad (2)$$

The subscripts of V denote differentiation with respect to the corresponding variable. This equation has two pairs of roots, given respectively, in the principal order in the small parameters Ω^2 and k^2 , by

$$\omega_{1,2}^2 = V_{\Phi\Phi} + \frac{k^2}{2}, \quad (3)$$

$$\omega_{3,4}^2 = \frac{P+4}{2(P+2)} k^4 - \frac{P k^2}{2 V_{\Phi\Phi} + k^2} \left[(P+4)(V_{PP} V_{\Phi\Phi} - V_{P\Phi}^2) + (P+2) k^2 V_{PP} - \frac{k^4}{2(P+2)} \right]. \quad (4)$$

Spin waves against the background of the equilibrium state ($\beta = 0$) have already been considered in Refs. 3 and 5. The solution (3) corresponds to a longitudinal mode and the difference from the case $\beta = 0$ consists only in the fact that the frequency of the longitudinal oscillations $\Omega = V_{\Phi\Phi}^{1/2}$ is now a function of the angle β . A more substantial difference arises in the second mode (4), and manifests itself particularly clearly in the case $k^2 \ll V \sim \Omega^2$, and then

$$\omega_{3,4}^2 = -\frac{1}{2} P (P + 4) k^2 \frac{D}{V_{\Phi\Phi}}, \quad (5)$$

where $D = V_{pp}V_{\Phi\Phi} - V_P^2$. That is to say, the spin waves have in this case a linear rather than quadratic dispersion, and their velocity depends substantially on the form of the energy of the spin-orbit interaction V .

In a large volume of liquid He^3 -B, the vector \mathbf{n} is oriented parallel to the magnetic field, and in this case we have

$$V = \frac{2\Omega^2}{15} \left[P + \frac{1}{2} + (P + 2) \cos^2 \Phi \right]^2$$

while at $P > -5/4$ we have $D = 0$. That is to say, the indicated linear term is absent, but appears at $P < -5/4$, when $D/V_{\Phi\Phi} = 16\Omega^2/15$.

Using a system of plane-parallel plates it is possible to orient the vector \mathbf{n} at the initial instant of time at different angles to the external field.⁶ The solutions describing the homogeneous precession of the magnetization for this case are given in Appendix B of Ref. 2. Using formula (B1) of this article, we find that in the region numbered 1 in the Appendix we have $D/V_{\Phi\Phi} = -(25/32)\Omega^2 \sin^4 \chi$, where χ is the angle between \mathbf{n} and \mathbf{S} . The sign of $\omega_{3,4}^2$ in this case is determined by the sign of $D/V_{\Phi\Phi}$, i.e., $\omega_{3,4}^2 < 0$ and the homogeneous precession is unstable. This instability is essential for the understanding of the mechanism of magnetization relaxation to the equilibrium state, and is worthy of the promptest experimental investigation.

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