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ELECTROMAGNETIC FORM FACTOR FOR THE PION IN THE DUAL RESONANT MODEL

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We show in the present paper that the form factors proposed in [1 - 4] can be obtained from the unitarity relation for the matrix element of the electromagnetic current

$$\mathrm{Im} < p_1, p_2|J_\mu(0)|0> = \frac{(2\pi)^4}{2}\sum_n \delta^4(p_1+p_2-p_n) < p_1, p_2|T^+|n><n|J_\mu(0)|0> \qquad (1)$$

if we neglect in it all the intermediate states except the two-pion one. Here T is the scattering matrix, and the pion isovector form factor is determined by the relation

$$< p_1, p_2|J_\mu(0)|0> = (p_1 - p_2)_\mu F(s), \quad s = (p_1+p_2)^2. \qquad (2)$$

Within the framework of this approximation, the form factor F(s) can be written in the form

$$F(s) = \Phi(s)\Omega(s), \qquad (3)$$

where $\Phi(s)$ is an entire function and $\Omega(s)$ is the Omnes function

$$\Omega(s) = \exp\left\{\frac{s}{\pi}\int_{4\mu^2}^{\infty}\frac{ds'\delta(s')}{s'(s'-s-i\epsilon)}\right\}. \qquad (4)$$

The equality follows from the fact that $\Phi(s) = F(s)/\Omega(s)$ is analytic in any finite part of the complex s plane. The function $\Omega(s)$ depends on the phase of the form factor $\delta(s)$, which is determined by the elastic-unitarity relation

$$\mathrm{Im}\,F(s) = F(s)f_1^*(s)\theta(s-4\mu^2) = F^*(s)f_1(s)\theta(s-4\mu^2). \qquad (5)$$

Here $f_1(s)$ is the partial amplitude of $\pi\pi$ scattering with ℓ, I = 1. The last equation in (5) follows from the fact that the imaginary part of the form factor is real if s is real. To determine the phase $\delta(s)$, we consider the partial amplitude $H_1(s) = (16\pi\sqrt{s}/k)f_1(s)$ in the Veneziano model [5, 6]

$$H_1(s) = \frac{1}{2}\int_{-1}^{1} dz P_1(z) T^{I=1}(s,t) = \frac{1}{2}\int_{-1}^{1} dz P_1(z)[A(s,t) - A(s,u)]$$
$$= \int_{-1}^{1} dz\, P_1(z) A(s,t), \qquad t = \frac{s-4\mu^2}{2}(z-1), \qquad (6)$$

where

$$A(s,\ t) = -\ 2g^2_{\rho\pi\pi}\ \frac{\Gamma(1-\alpha_s)\Gamma(1-\alpha_t)}{\Gamma(1-\alpha_s-\alpha_t)}$$

α_s and α_t are the linear ρ trajectories in the s and t channels, $\alpha_s = \alpha(s) = a + bs$; $\alpha(0) = 1/2$.

As noted in [7], the dual resonant amplitude A(s, t) satisfies the unitarity relation in second order in the coupling constant.

Using the representation [8]

$$\frac{\Gamma(1+a)\Gamma(1+b)}{\Gamma(1+a+b)} = \prod_{k=1}^{\infty} \frac{k(a+b+k)}{(k+a)(k+b)} \tag{7}$$

we expand the amplitude $H_1(s)$ in the form of an infinite product

$$H_1(s) = \int_{-1}^{1} dz P_1(z) \prod_{J=1}^{\infty} \frac{J(J-\alpha_s-\alpha_t)}{(J-\alpha_s)(J-\alpha_t)}\ . \tag{8}$$

By virtue of the microcausality condition, the amplitude $H_\ell(s)$ should be taken to mean the limit of $H_\ell(s + i\varepsilon)$ as $\varepsilon \to +0$. This means that the pole factors

$$f_J = \frac{1}{J-\alpha_s-i\epsilon} = \frac{1}{b(m_J^2-s-i\epsilon)} = \frac{1}{|J-\alpha_s-i\epsilon|}\ e^{i\delta_J} \tag{9}$$

should have at s > 0 a phase equal to

$$\delta_J = \operatorname{arc\,tg} \frac{\epsilon}{J-\alpha_s}\ . \tag{10}$$

The poles in the t channel lead to the occurrence in $H_\ell(s)$ of cuts $-\infty \leq s \leq 4\mu^2 - m_J^2$, as can be readily verified by expanding A(s, t) in the series $\sum_J \bar{C}_J(s)/(J - \alpha_t)$ and integrating with respect to z. Thus, for a positive s we can write $H_1(s)$ in the form

$$H_1(s) = e^{i\delta_1(s)} h_1(s),\ \text{где}\ \ h_1(s) = \int_{-1}^{1} dz P_1(z) \prod_{J=1}^{\infty} \frac{J(J-\alpha_s-\alpha_t)}{|J-\alpha_s|(J-\alpha_t)} \tag{11}$$

is a real alternating-sign function and

$$\delta_1(s) = \sum_{J=1}^{\infty} \delta_J(s)\ . \tag{12}$$

From (5) it follows directly that the phase of the form factor $\delta(s)$ is equal to the phase $\delta_1(s)$. In the limit as $\varepsilon \to +0$ we obtain from (12) and (10)

$$\delta(s) = \pi \sum_{J=1}^{\infty} \theta(\alpha_s - J) = \pi \sum_{J=1}^{\infty} \theta(s - m_J^2). \tag{13}$$

Choosing α_s as the integration variable, we introduce the Omnes function with two subtractions

$$\Omega(\alpha_s) = \exp\left\{\frac{\alpha_s^2}{\pi}\int_{\alpha(4\mu^2)}^{\infty}[\alpha_s'^2(\alpha_s' - \alpha_s - i\epsilon)]^{-1}\delta(\alpha_s')\,d\alpha_s'\right\}. \tag{14}$$

Taking (13) into account, we rewrite the function $\Omega(\alpha_s)$ in the form of a product

$$\Omega(\alpha_s) = \prod_{J=1}^{\infty}\Omega_J(\alpha_s), \quad \Omega_J(\alpha_s) = \exp\left\{\alpha_s^2\int_J^{\infty}\frac{d\alpha_s'}{\alpha_s'^2(\alpha_s' - \alpha_s - i\epsilon)}\right\}. \tag{15}$$

Carrying out the integration in (15), we obtain

$$\begin{aligned}\Omega_J(\alpha_s) &= \exp\left\{\ln\left|\frac{J}{J-\alpha_s}\right| + i\pi\theta(\alpha_s - J) - \frac{\alpha_s}{J}\right\}\\ &= \exp\left\{\ln\frac{J}{J-\alpha_s} - \frac{\alpha_s}{J}\right\} = e^{-\alpha_s/J}\left(1-\frac{\alpha_s}{J}\right)^{-1}\end{aligned} \tag{16}$$

whence

$$\Omega(\alpha_s) = \prod_{J=1}^{\infty} e^{-\alpha_s/J}\left(1-\frac{\alpha_s}{J}\right)^{-1} = \Gamma(1-\alpha_s)\,e^{-\alpha_s C} \tag{17}$$

C is the Euler constant. The possible form of the entire function $\Phi(s)$ can be established only by using additional physical requirements, primarily the condition for normalization at $s = 0$ and convergence at infinity. $F(s) \to 0$ as $|s| \to \infty$. These conditions are not contradicted by a choice of $\Phi(s)$ leading to a form factor of the type

$$F(s) = \beta\frac{\Gamma(1-\alpha_s)}{\Gamma(\lambda-\alpha_s)}. \tag{18}$$

At $\lambda = 5/2$ we obtain perfectly good agreement with the experimental data at a ρ-meson width $\Gamma_\rho = 130$ MeV. The function

$$F(s) = \frac{1}{\sqrt{\pi}}\frac{\Gamma(1-\alpha_s)}{\Gamma(5/2-\alpha_s)} \tag{19}$$

gives $|F(m_\rho^2)|^2 = 56$ and $(r_\pi^2)^{1/2} = 0.7 \times 10^{-13}$ cm. The experimental values [9] are $|F(m_\rho^2)|^2 = 58.3 \pm 5.6$ and $(r_\pi^2)^{1/2} = 0.8 \pm 10;\ 0.86 \pm 0.09$ [sic!].

We note that in [10] the parameter λ in (18) was determined from the equality of the phases of the form factor and of the partial amplitude. However, such a method is inconsistent, since the phase of the entire function, which appears on going over into region of complex s, is not connected with the phase of the partial scattering amplitude.

Thus, our calculation of the Omnes function shows that the form factors obtained in [1 - 3] satisfy the elastic unitarity relation.

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"FORBIDDEN" CROSS-RELAXATION TRANSITIONS AND THEIR ROLE IN THE DYNAMICS OF STATIONARY ENDOR

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The frequencies of the electron-nuclear double resonance (ENDOR) are due to the hyperfine interaction (HFI) of the electron of the paramagnetic center with the lattice nuclei. If the electron spin of the center is $S = 1/2$, then each nucleus gives two ENDOR frequencies for radio frequency (RF) transitions with $M = 1/2$ and $M = -1/2$, respectively (M is the projection of $\vec{S}$ on the external magnetic field $\vec{H}$). In the simplest case these frequencies [1, 2] are

$$\nu_{\mp 1/2} = \left|\frac{1}{2}a + \frac{1}{2}b(3\cos^2\theta - 1) \pm \nu_n\right|. \tag{1}$$

where a and b are the constants of the isotropic and anisotropic HFI with the nucleus, ν_n is the Larmor frequency of the nucleus, and θ is the angle between the axial axis of the HFI and $\vec{H}$. The frequencies $\nu_{-1/2}$ and $\nu_{1/2}$ are customarily called the summary and differential frequencies. We confine ourselves to a very simple model, which retains the main features of the real multilevel system, namely, we consider a paramagnetic center with $S = 1/2$ and two nuclei with spins $I_1 = I_2 = 1$. Let $a_1 \gg a_2$ and $|b_i| \ll a_i$ ($i = 1, 2$). Figure 1 shows the energy levels of our system; the frequencies (1) are determined by the selection rules $\Delta M = 0$, $\Delta m_1 = \pm 1$, $\Delta m_2 = 0$, $\Delta m_1 = 0$, $\Delta m_2 = \pm 1$, where m_1 is the projection of $\vec{I}_1$ on $\vec{H}$ and m_2 is the projection of $\vec{I}_2$ on the quantization axis $n(M)$, which does not coincide with $\vec{H}/H$ if

$$\left|\frac{1}{2}a_2 - \nu_{n2}\right| \sim b_2$$

(see [2]), and depends on M.

Great interest attaches to the dynamics of the stationary ENDOR [3, 4] for the study and separation of different relaxation processes in multilevel systems. One of the questions that has remained unanswered for ten years is that of the difference between the intensities δ of the ENDOR signals of paramagnetic centers at the summary and difference frequencies: $\delta_c \neq \delta_p$ (in particular, for F centers in a number of alkali-halide crystals, see [3]). It was shown experimentally in [5] that the effect $\delta_c \neq \delta_p$ is due to the inclination of the quantization axis (IQA) of the nuclei of one of the coordination spheres of the