[6] M.G. Smoes, Nucl. Phys. B20, 237 (1970).
[7] P. Depommier, J. Heintze, C. Ruboia, and V. Soergel, Phys. Lett. 7, 285 (1963).
[8] P. Dennery and H. Primakoff, Phys. Lett. 6, 67 (1963).
[9] S.A. Pikin and Yu.N. Kharkats, Yad. Fiz. 1, 291 (1965) [Sov. J. Nuc. Phys. 1, 207 (1965)].
[10] D.Yu. Bardin, S.M. Bilen'kii, G.V. Mitsel'makher, and N.M. Shumeiko, JINR Preprint R2-5520, Dubna, 1970.

DUAL AMPLITUDE FOR PROCESSES WITH PARTICIPATION OF PHOTONS

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There is no doubt that a very important problem in elementary particle physics is the construction of the dual amplitude containing besides the hadrons photons and (or) leptons. In this paper we attempt to obtain a dual amplitude for the process N mesons + γ. The analysis was carried out in terms of the approach [1, 2] based on the parton model. The obtained amplitude is compared with the Veneziano amplitude [3], which was used earlier [4] to describe the processes $\gamma\pi \to 2\pi$. One of the interesting properties of this approach, in our opinion, is the possibility of carrying out an analogy with the vector dominance model (VDM).

We assume that the partons are vector charged particles. We choose the following interaction Lagrangians: $\lambda\varepsilon_{\alpha\beta\gamma\delta}\phi_\alpha\phi_\beta\phi_\gamma\phi_\delta$ for the parton-parton field, $e\varepsilon_{\alpha\beta\gamma\delta}A_\alpha(\phi_\beta\partial_\gamma\phi_\delta^* - \partial_\gamma\phi_\beta\phi_\delta^*) + eA_\alpha\phi_\alpha$ for the parton and electromagnetic field, and $\lambda\partial_\mu M\phi_\nu\phi_\alpha\phi_\beta\varepsilon_{\mu\nu\alpha\beta}$ for the parton field and the field of the spinless mesons; here ϕ_α, A_α, and M are respectively the parton, electromagnetic, and meson fields. If, following [2], we write the amplitude corresponding to Fig. 1 and then expand $\ln\{P[(X^2)]\}$ ($P[(X^2)]$ is the parton propagator) near $X^2 \sim 0$, then going to the limit as $n \to \infty$, where n is the number of vertices of the diagram of Fig. 1, we obtain

$$M = f_{ij}\epsilon_i p_j A,$$
$$A = C\delta\left(\sum_{i=1}^{N} k_i + p\right)\exp\{-L(z_1, \ldots, z_N, z_\nu)\}, \tag{1}$$

where $L(z_1, \ldots, z_N, z_\nu)$ is the Lagrangian of a liquid flowing through a flat disk of unit radius with apertures at the points z_i and z_ν ($z_i = \exp[i\theta_i]$ $z_\nu = \rho\exp[i\theta]$, $\theta_i \geq \theta_{i+1}$), the liquid current being k_i in aperture z_i and p in z_ν; $f_{ij} = - f_{ji}$. The latter condition guarantees gauge invariance of the amplitude. The Lagrangian L will be represented in the form

$$L(z_1, \ldots, z_N, z_\nu) = \sum_{i<j} R_{ij} k_i k_j + \sum_{j=1}^{N} R_{j\nu} k_j p, \tag{2}$$

where $R_{\alpha\beta}$ is the resistance of the disk between the points z_α and z_β, equal to

$$R_{\alpha\beta} \sim \ln|z_\alpha - z_\beta| \quad \text{при } z_\alpha \to z_\beta. \tag{3}$$

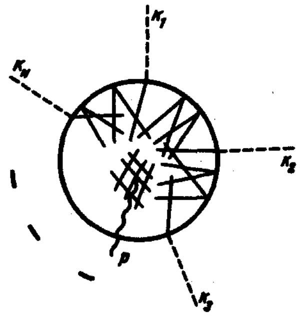

Fig. 1

From (1) - (3), after integrating over all the possible placements of z_i and z_ν ($\theta_{i+1} \leq \theta_i$), we obtain:

$$A(s_{\alpha\beta}) = C \int_0^{2\pi} d\theta \int_0^1 d\rho\rho \int_0^{2\pi} d\theta_1 \int_0^{\theta_1} d\theta_2 \dots \int_0^{\theta_{N-1}} d\theta_N \times \prod_{i=1}^{N} \left|z_\nu - z_i\right|^{-\alpha' s_i} \prod_{i<j} \left|z_i - z_j\right|^{-\alpha' - s_{ij}}, \tag{4}$$

where

$$s_i = 2pk_i, \quad s_{ij} = 2k_i k_j.$$

We note that the VDM has a very simple treatment in such an approach. To this end, we transform the photon-related cell of our diagram in the manner shown in Fig. 2. After going through the foregoing procedure, we obtain the amplitude (4) multiplied by the "vector meson" propagator $\{P[(p^2)]\}^2$ = const, i.e., we obtain (4) with a redefined constant C.

As shown in [3, 4], the process $\gamma\pi \to 2\pi$ can be described by the following dual amplitude:

$$A(s,t) = \int_0^1 du_3^3 \int_0^1 du_3^2 (u_3^3)^{-\alpha_\omega(0)} (1 - u_3^3)^{-\gamma_1} \times \frac{1}{1 - u_3^3 u_3^2} (u_3^2)^{-\alpha_\rho(s)} \left(\frac{1 - u_3^2}{1 - u_3^3 u_3^2}\right)^{-\alpha_\rho(t)},$$

where γ_1 is an unknown trajectory in the spurion channel, and to reconcile the formula with experiment it is necessary to put $\gamma_1 = 0$. We express the integration variables in (5), in analogy with [5], in terms of the five variables z_1, z_2, z_3, z_4, and z:

$$\begin{cases} u_3^1 = \dfrac{1 - u_3^2}{1 - u_3^3 u_3^2} = \dfrac{(z_4 - z_1)(z_3 - z_2)}{(z_4 - z_2)(z_3 - z_1)} \\[2ex] u_3^2 = \dfrac{(z - z_2)(z_4 - z_3)}{(z - z_3)(z_4 - z_2)} \\[2ex] u_3^3 = \dfrac{(z - z_1)(z_4 - z_2)}{(z - z_2)(z_4 - z_1)} \end{cases} \tag{6}$$

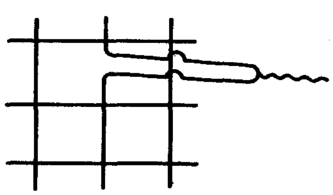

Fig. 2

If we now put $z_j = \exp[i\theta_j]$ and $z = \rho z_1$, then we obtain from (5) an amplitude quite similar to (4):

$$A(s,t) = C \int_0^1 d\rho \int_0^{2\pi} d\theta \int_0^{\theta} d\theta_2 \int_0^{\theta_2} d\theta_3 \int_0^{\theta_3} d\theta_4 \left(\frac{1-\rho}{\rho}\right)^{-\alpha_\omega(0)}$$

$$\times \prod_{i=1}^{3} |z - z_i|^{-\alpha_\omega(0)} |z - z_2|^{-\alpha_\rho(s)} |z - z_3|^{-\alpha_\rho(u)} |z - z_4|^{-\alpha_\rho(t)}$$

$$\times |z_2 - z_3|^{-\alpha_\rho(t)} |z_2 - z_4|^{-\alpha_\rho(u)} |z_3 - z_4|^{-\alpha_\rho(s)}, \tag{7}$$

where

$$C = \int_0^{2\pi} d\theta_1 \int_0^{\theta_1} d\theta_3 \int_0^{\theta_3} d\theta_4 \{|z_1 - z_3| |z_1 - z_4| |z_3 - z_4|\}^{-1}.$$

In the derivation of (7) it was assumed that the following condition is superimposed on the ρ and ω trajectories:

$$\alpha_\rho(s) + \alpha_\rho(t) + \alpha_\rho(u) = \alpha_\omega(0) + 1. \tag{8}$$

This condition, in conjunction with the condition $\alpha_\rho(s) + \alpha_\rho(t) + \alpha_\rho(u) = 1/2$ on the ρ trajectory, predicts $\alpha_\omega(0) = 1/2$, which agrees with experiment. We note that if we assume that $\alpha'_\omega = \alpha'_\rho$, then we obtain the relation $m^2_\omega = m^2_\rho + m^2_\pi$, which agrees well with experiment.

The difference between formulas (4) and (7) lies only in the factor

$$\left(\frac{1-\rho}{\rho}\right)^{-\alpha_\omega(0)} \prod_{i=1}^{3} |z - z_i|^{-\alpha_\omega(0)},$$

which is replaced in (4) by the "vector meson" propagator $\{P[0]\}^2$, and the factor

$$\prod_{i=1}^{3} |z - z_i|^{-\alpha'_\rho m^2_\pi}$$

which is missing from (7).

We have thus shown that for the simplest non-hadronic dual amplitudes the model gives reasonable results. One can hope that the parton model can serve as a good tool for constructing more complicated non-hadronic amplitudes. The advantage of such an approach consists, in our opinion, in the fact that it does not require any physically new assumptions, other than those made in the case of purely hadronic interactions.

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[1] H.B. Nielsen, NORDITA Preprint, 1970; D.B. Fairlie and H. Nielsen, Nucl. Phys. B21 (1970); H. Nielsen and P. Olesen, Phys. Lett. 32B (1970).
[2] B. Sakita and M.A. Virasoro, Phys. Rev. Lett. 24, 1146 (1970); B. Sakita, Preprint ITF-70-99 (1970).
[3] F. Cooper, Phys. Rev. 1D, 1140 (1970).
[4] G. Murtasa and A.M. Harum-Ar-Rashid, Phys. Rev. 2D, 236 (1970).
[5] Z. Koba and H. Nielsen, Nucl. Phys. B12, 517 (1970).