Anomalous magnetic relaxation in superfluid helium-3

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The change of the character of the relaxation of the magnetization in superfluid helium-3 when the magnetization is inclined at large angles is attributed to a transition from an internal stationary Josephson effect to a nonstationary one. The dependence of the critical angle of inclination on the temperature and on the magnetic field is determined. Comparison is made with experiment.

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In recent experiments, Webb[1] observed in superfluid phases of He^3 a change in the character of the relaxation of the magnetization when the magnetization was inclined away from equilibrium by an angle larger than a certain critical value. It is shown in the present article that this phenomenon can be explained if account is taken of the influence of the inhomogeneity of the constant magnetic field $\mathbf{H}_0$. The field inhomogeneity causes the rate of precession of the magnetization at different points of space to be different, as a result of which the magnetization gradients increase with time. The

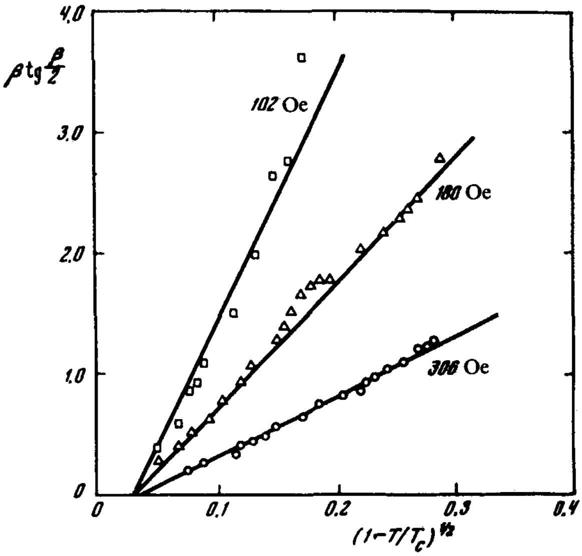

FIG. 1. Points–experimental data of Webb.[1] The straight lines were drawn through the points. The ratio of the slopes of the lines for 180 and 306 Oe agree well with the $H_0^{-3/2}$ law. The intercepts of the lines with the abscissa axis correspond approximately to the temperatures of the transition into the A_1 phase.

diffusion spin fluxes caused by these gradients can be cancelled out by "outflow" via the spin–orbit interaction V only so long as the spin influx does not exceed the maximum outflow for a given V, after which a nonstationary regime sets in, similar to the transition in Josephson junctions.

For a quantitative description of these processes, we use the Leggett system of equations in the strong-field approximation in the form given in Ref. 2, and add to these equations terms that contain $(\nabla\alpha)^2$, where α is the angle that describes the precession of the magnetization. The aforementioned gradient is proportional to the time and at large values of the time its square is the principal one of the terms with the spatial derivatives.[3] We shall assume that the field H_0 has a constant derivative H_0/l along the y direction and will designate differentiation with respect to y by a prime, and then $\alpha' = -\omega_L t/l$. As a result, the equations which we need, in the dimensionless form and in the notation of Ref. 2, are

$$\dot{S} = -\frac{\partial V}{\partial \Phi} - \frac{DS}{\omega_L}\left(\frac{t}{l}\right)^2 \sin^2\beta, \tag{1}$$

$$\dot{\Phi} = S - 1, \tag{2}$$

$$P = \frac{DS}{\omega_L}\left(\frac{t}{l}\right)^2 \sin^2\beta. \tag{3}$$

Here D is the spin-diffusion coefficient. Equations (1) and (2) describe longitudinal oscillations of the magnetization with characteristic frequency $\Omega \sim 10$ kHz. In this scale, P and α' vary slowly and we can use their instantaneous values in the analysis of the system (1) and (2). The system has stationary solutions $S = 1$ and $\Phi = \Phi_0$, where Φ_0 is the root of the equation

$$\frac{\partial V}{\partial \Phi} + \frac{D}{\omega_L}\left(\frac{t}{l}\right)^2 \sin^2\beta = 0, \tag{4}$$

so long as (4) has solutions. Substituting $S = 1$ in (3) and recognizing that $P = S \times(\cos\beta - 1)$, we find that in this case the relaxation follows the law

$$\operatorname{tg}\frac{\beta}{2} = \left(\operatorname{tg}\frac{\beta_0}{2}\right)\exp\{(t_0^3 - t^3)/\tau^3\}, \tag{5}$$

where $\tau^3 = 3l^2/D\omega_L^2$ in dimensional units.

At a certain $t = t_c$ Eq. (4) ceases to have roots and a transition to nonstationary regime takes place. It is natural to assume in the experiments of Ref. 1 that the critical initial inclination angle of the magnetization, β_c was the angle at which the solutions of (4) vanished at the instant of termination of the process of magnetization rotation. Using the fact that the angle of rotation of the magnetization is proportional to the time of rotation, i.e., $\beta_c = \omega_L t_c H_1/2H_0$ (H_1 is the amplitude of the alternating rotating field), and also the explicit form of the potential V for the A phase (see formula (18) of Ref. 2), we obtain an equation for β_c in the A phase:

$$\left(\beta_c \operatorname{tg}\frac{\beta_c}{2}\right)^2 = \frac{\Omega_A^2}{8D\omega_L}\left(\frac{H_1 l}{2H_0}\right)^2. \tag{6}$$

Formula (6) is compared in Fig. 1 with the experimental data.[1] The agreement is good for fields 180 and 306 Oe, and worse for $H_0 = 102$ Oe. It must be borne in mind, however, that the vanishing of the roots of Eq. (4) is a sufficient condition for the transition to the nonstationary solution. In the A phase it is possible to obtain more complete information on the solutions of the system (1) and (2) by using the fact that in this case the system reduces to an equation that describes the motion of a pendulum with friction under the influence of a constant moment:

$$\ddot{\theta} + \sin\theta + \lambda\dot{\theta} + \mu = 0, \tag{7}$$

where

$$\lambda = (D/\Omega_A)\left(2a'\sin\frac{\beta}{2}\right)^2, \quad \mu = (8D\omega_L/\Omega_A^2)\left(a'\operatorname{tg}\frac{\beta}{2}\right)^2 .$$

This equation was investigated (see Ref. 4) and it was shown that the transition to the stable nonstationary solution can occur both at $\mu > 1$, which yields criterion (6), and at $\mu < 1$, $\lambda < \lambda_0$, where λ_0 is a certain function of μ. Since the numerator of μ contains H_0 raised to one power higher than in λ, the violation of the condition $\lambda > \lambda_0$ with decreasing field H_0 becomes more probable. The choice between the stationary and periodic solutions at $\lambda < \lambda_0$ is determined by the initial conditions.

The following qualitative remarks can be made concerning the relaxation of the magnetization after the transition to the nonstationary regime. The average magnetization outflow ensured by the interaction V is less than the maximum possible for the given V, and can therefore not offset the diffusion influx of magnetization, and the influence of V can be disregarded in a rough estimate in those cases when appreciable diffusion fluxes are present. The relaxation process can therefore be visualized as occurring in two stages. Within a time of the order of τ the diffusion equalizes the magnetization gradients on account of relaxation of the angle β at a fixed projection of the magnetization on the direction of $\mathbf{H}_0$. At small angles β the diffusion becomes ineffective and further relaxation of the longitudinal component of the magnetization is due to the slower "internal" relaxation mechanism,[5] as is apparently observed in fact in the experiment.

In the B phase, the potential V has a more complicated form and a complete investigation of the conditions for the transition to the nonstationary regime was not carried out. The use of the simple criterion (4) does not lead to results that agree with experiment. The qualitative similarity between the phenomena in the A and B phases is evidence, however, that the physical cause of the observed phenomena is the same in both phases.

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