COHERENT BREMSSTRAHLUNG OF ELECTRONS AND POSITRONS OF ULTRAHIGH ENERGY IN CRYSTALS

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The bremsstrahlung of electrons and positrons in the energy region $\varepsilon_1 \gtrsim 1$ GeV has a coherent character at small inclination angles θ of the primary beam to the crystal axis. The theory of the effect in the first Born approximation was given in [1 - 3]. In this approximation, there is no distinction between the radiation of electrons and positrons.

We wish to show that at sufficiently small angles θ the coherent effect increases the relative contribution of the second and higher Born approximations, and the expansion parameter becomes not the usual Ze^2 but $Ze^2/\varepsilon_1 a\theta^2$, where ε_1 is the electron energy and $\underline{a}$ is the lattice constant. This leads to an appreciable difference between the radiation of electrons and positrons at small angles θ even in crystals of light elements.

The cross section with allowance for the first and second Born approximations is given by

$$d\sigma = Z^2\alpha^3 \frac{p_2}{p_1} \frac{\pi}{a^3} \sum_g \delta(q - g)[S_1 V^2(g) + + 2\,\mathrm{Re}\,v(g)a^{-3} \sum_{g_1} S_2 v(g_1) v(g - g_1)]\omega\, d\omega\, dO dO_2. \tag{1}$$

Here S_1 and S_2 are the traces of the products of the matrices constructed in the usual manner from the Feynman diagrams [4]; $v(\vec{g}) = (\vec{g}^2 + \rho^{-2})^{-1}$ is the Fourier component of the screened Coulomb potential, ρ is the screening radius, $\vec{g}$ and $\vec{g}_1$ are the reciprocal-lattice vectors, and $\vec{q} = \vec{p}_1 - \vec{p}_2 - \vec{k}$ is the momentum transferred to the crystal. S_1 and S_2 contain denominators of the type

$$\kappa = 2p_1 g - g^2 = 2\epsilon_1 g_\ell (1 - g^2/2\epsilon_1 g_\ell),$$

$$\tau = 2p_2 g_1 + g_1^2 = 2\epsilon_2 g_{1\ell}(1 + g_1^2/2\epsilon_2 g_{1\ell} + n_\perp g_1/g_{1\ell}),$$

where g_ℓ is the projection of g on p_1. From the kinematics of the process it follows [1 - 3] that $g_\ell \geq \delta = \omega m^2/2\varepsilon_1\varepsilon_2$.

We consider for simplicity a cubic lattice and assume that the momentum $\vec{p}_1$ lies in crystal plane xz (θ is the angle between $\vec{p}_1$ and the crystal axis z). The main coherent contribution to the cross section (1) are then made by the reciprocal-lattice vectors with $g_z = g_{1z} = 0$. For these, $g_\ell = \theta g_x$ and $g_{1\ell} = \theta g_{1x}$, leading to the appearance of factors θ^{-2} in S_1 and θ^{-3} in S_2.

For the values of $\vec{g}$ and $\vec{g}_1$ which make the main contribution to the cross section, the second and third terms in the parentheses in the expressions for κ and τ are small, making it possible to simplify greatly the expressions for S_1 and S_2, and in the case of S_2 there is separated an additional factor θ^{-1}. Integrating further with respect to $dOdO_2$, we obtain

$$d\sigma = Z^2 a^3 \frac{p_2}{p_1} \frac{(4\pi)^2}{a^3} \sum_{g_x, g_y} [\bar{S}_1 v^2(\mathbf{g}) + + 2v(\mathbf{g}) a^{-3} \sum_{g_{1x}, g_{1y}} \bar{S}_2 v(\mathbf{g}_1) v(\mathbf{g} - \mathbf{g}_1)] \delta \frac{d\omega}{\omega} , \qquad (2)$$

where

$$\bar{S}_1 = \frac{\epsilon_1}{\epsilon_2} \frac{g_\perp^2}{g_\ell^2} \left[1 + \left(\frac{\epsilon_2}{\epsilon_1}\right)^2 - 4\frac{\epsilon_2}{\epsilon_1} \frac{\delta}{g_\ell} \left(1 - \frac{\delta}{g_\ell}\right)\right] ,$$

$$\bar{S}_2 = 4\pi \frac{Z a}{\epsilon_1 a \theta^2} \left[1 - 2\frac{g_x^2}{g_\perp^2} \frac{\delta}{g_\ell} \left(1 - \frac{\delta}{g_\ell}\right)\right] \frac{\mathbf{g}_1 \mathbf{g}_2}{g_{\ell x}^2} \frac{g^2 + g_1^2 - g_2^2}{g_\ell^2} ,$$

$$\mathbf{g}_2 = \mathbf{g} - \mathbf{g}_1 , \quad g_\perp^2 = g_x^2 + g_y^2 , \quad \omega/\epsilon_1 << 1.$$

The calculation of the sums entering into $d\sigma$ in the real case of a three-dimensional crystal leads to difficulties. We shall therefore consider a model that differs from a real crystal in the averaging of the potential over the y axis. This averaging reduces to imposition of the conditions $g_y = g_{1y} = 0$ in the sums (2).

The sum over g_{1x} can be replaced with good accuracy by an integral. Assuming that $\omega/\varepsilon_1 << 1$, we obtain as a result

$$d\sigma = Z^2 a^3 \frac{(4\pi)^2}{a^3} \sum_{g_x \geq \delta\theta^{-1}} \frac{\exp(-A g_x^2)}{(g_x^2 + \rho^{-2})^2} \frac{2\delta}{\theta^2} \times \left[1 - \frac{2\delta}{g_\ell}\left(1 - \frac{\delta}{g_\ell}\right)\right] \left(1 + \frac{2\pi Z a}{\epsilon_1 a \theta^2} \frac{\rho}{a} \frac{3}{4 + \rho^2 g_x^2}\right) \frac{d\omega}{\omega} , \qquad (3)$$

where we have introduced the factor $\exp(-A g_x^a)$, which takes into account the thermal motion of the atoms of the crystal [1 - 3].

In this expression, in the case of positrons $Z > 0$, and in the case of electrons $Z < 0$, and therefore the second Born approximation increases the cross section for the radiation of positrons compared with that of electrons. With decreasing angle θ, this effect increases like θ^{-2}. At very small θ, when the contribution of the second approximation becomes comparable with the contribution of the first, formula (3) ceases to be valid, and it is necessary to take into account the higher Born approximations.

We note that the incoherent part of the cross section does not lead to a noticeable difference between the radiation of the electrons and the positrons.

For a real three-dimensional crystal there appears in the expression for the second Born approximation a factor $(a/\rho)^2 >> 1$ in addition to (3), and the relative contribution of the second approximation becomes of the order of $Z\alpha/\varepsilon\rho\theta^2$.

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DISTRIBUTION OF ENERGY LOSSES FOR HEAVY POSITIVE PARTICLES IN A SINGLE CRYSTAL

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The phenomenon of channeling of charged particles was discussed theoretically in a number of papers [1], but nonetheless the distribution function of the ionization losses of the energy under conditions of channeling has not yet been calculated. The purpose of the present article is to call attention to the fact that in the case of channeling of heavy particles there exists an approximation in which the inelastic processes can be described in an exceedingly simple manner.

Indeed, taking into account the smallness of the characteristic angles of the scattering of the heavy particle by an electron compared with the characteristic angles of elastic scattering of a heavy particle by a nucleus, it can be assumed that the scattering by an electron does not change the direction of motion of the heavy particle, and changes only its energy.

In this approximation, inelastic scattering does not change the distribution of the heavy particles over the cross section of the beam. Therefore, in the kinetic equation for the distribution function of the particles with respect to the energy loss Δ and the coordinates x and $\vec{r}_\perp$, the transverse coordinates $\vec{r}_\perp$ play the role of parameters:

$$\frac{\partial f(x, r_\perp, \Delta)}{\partial x} = \int_0^\infty d\epsilon\, w_E(\epsilon, n_\perp)[f(x, r_\perp, \Delta - \epsilon) - f(x, r_\perp, \Delta)], \tag{1}$$

In the case of small losses ($\Delta \ll E$) the probability of loss of energy ε by a particle per unit path $w_E(\varepsilon, \vec{r}_\perp) \simeq w_{E_0}(\varepsilon, \vec{r}_\perp)$, and the solution of (1) takes the form

$$f(x, r_\perp, \Delta) = (2\pi i)^{-1} \int_{-i\infty+\sigma}^{+i\infty+\sigma} dp \exp\{-p\Delta - \int_0^\infty d\epsilon\, w_{E_0}(\epsilon, r_\perp)[1 - e^{-p\epsilon}]\}. \tag{2}$$

The final particle-energy distribution function is obtained by averaging (2) over the spatial distribution of the transverse coordinates of the particles $W(\vec{r}_\perp)$ due to the elastic scattering

$$f(x, \Delta) = \int_s d^2 r_\perp W(r_\perp) f(x, r_\perp, \Delta), \tag{3}$$

where the integration over the transverse coordinates has been reduced to integration over the transverse cross section of one unit cell.

2. In an amorphous medium $W(\vec{r}_\perp)$ does not depend on $\vec{r}_\perp$ and (3) coincides with the particle-energy distribution function in an amorphous medium [2], $f_{am}(x, \Delta)$. In a single crystal, the flux of positively charged particles moves in such a way that the number of units between the crystallographic