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HYDRODYNAMIC DRIFT-DISSIPATIVE INSTABILITIES OF A PLASMA WITH NON-UNIFORM TEMPERATURE

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Moiseev [1] was the first to note low-frequency drift instability in an inhomogeneous plasma with collisions, and called attention to the important role that longitudinal ion motion plays in its development. We show in this paper that allowance for the plasma temperature and viscosity inhomogeneities leads to the appearance of a number of new low-frequency instabilities.

In considering a plane plasma layer with initial density $n_0(x)$, electron and ion temperatures $T_{0e}(x)$ and $T_{0i}(x)$, and velocities $\vec{V}_{0e}(x)$ and $\vec{V}_{0i}(x)$, principal attention was paid to time intervals during which the initial distribution can be regarded as stationary. If $T_{0i} \neq T_{0e}$ this means that $\tau \approx 1/\omega \ll (m_i/m_e)\nu_e^{-1}$. When $T_{0e} = T_{0i}$ the limitations on the time intervals are weaker; they are connected with slow dissipative processes transverse to a strong ($\Omega_\alpha \gg \nu_\alpha$) magnetic field. We investigated the stability of the initial state of the plasma against potential perturbations $\vec{E} = -\vec{\nabla}\Phi$. For perturbations of the type $f(x)\exp(i\omega t + ik_y y + ik_z z)$ we can obtain the following system of linearized equations of hydrodynamics

$$\omega n^\alpha + \frac{k_y e\Phi}{m_\alpha \Omega_\alpha}\frac{\partial \ln n_0}{\partial x} + k_z V_z^\alpha n_0 = 0; \qquad (1)$$

$$V_z^e\left(im_e\omega + \frac{4}{3}\frac{k_z^2}{n_0}\eta_0^e\right) = -ik_z T^e(1 + S + a^e) - ik_z T_{0e}\frac{n^e}{n_0}\left(1 - a^e\frac{\partial \ln T_{0e}}{\partial \ln n_0}\right) + ik_z e\Phi; \qquad (2)$$

$$V_z^i\left(im_i\omega + \frac{4}{3}\frac{k_z^2}{n_0}\eta_0^i\right) = ik_z S T^e - ik_z T^i(1 + a^i) - ik_z T_{0i}\frac{n^i}{n_0}\left(1 - a^i\frac{\partial \ln T_{0i}}{\partial \ln n_0}\right) - ik_z e\Phi; \qquad (3)$$

$$T^e\left(\frac{3}{2}i\omega + k_z^2\chi_\parallel^e/n_0\right) = i\omega T_{0e}\frac{n^e}{n_0} - \frac{ik_y e\Phi}{m_e\Omega_e}T_{0e}\left(\frac{3}{2}\frac{\partial \ln T_{0e}}{\partial x} - \frac{\partial \ln n_0}{\partial x}\right) - 3\frac{m_e}{m_i}\nu_e(T^e - T^i); \qquad (4)$$

$$T^i\left(\frac{3}{2}i\omega + k_z^2\chi_\parallel^i/n_0\right) = i\omega T_{0i}\frac{n^i}{n_0} + \frac{ik_y e\Phi}{m_i\Omega_i}T_{0i}\left(\frac{3}{2}\frac{\partial \ln T_{0i}}{\partial x} - \frac{\partial \ln n_0}{\partial x}\right) + 3\frac{m_e}{m_i}\nu_e(T^e - T^i). \qquad (5)$$

In the derivation of these equations it was assumed that $T_{0e} = T_{0i} = T_0(x)$; for the transverse motion, terms containing $k_y^2\rho_\alpha^2$ were omitted[1)] (ρ_α = Larmor radius of the particles α; see

[2] for notation). Considering oscillation wavelengths much larger than the Debye radius of the plasma, we put $n^e = n^i = n$. Equations (1) - (5) then lead to the dispersion relation

$$[(1 - \beta_e)(1 - \beta_i) - 2i\frac{m_e \nu_e}{m_i \omega}(2 - \beta_e - \beta_i)][(1 - \frac{4}{3}i\frac{k_z^2 \eta_0^i}{\omega n_0 m_i})(1 + \frac{\omega_e}{\omega}) - 2\frac{k_z^2 v_{Ti}^2}{\omega^2}]$$

$$+ \frac{2}{3}\frac{k_z^2 v_{Ti}^2}{\omega^2}\left\{\beta_e[1 + \frac{\omega_e}{\omega}(1 - \frac{3}{2}\frac{\partial \ln T_0}{\partial \ln n_0})] - (1 - 4i\frac{m_e}{m_i}\frac{\nu_e}{\omega})[2 - 2s\frac{\omega_e}{\omega}(1 - \frac{3}{2}\frac{\partial \ln T_0}{\partial \ln n_0})]\right\}$$

$$+ (1 + S)\frac{\omega_e}{\omega}\frac{\partial \ln T_0}{\partial \ln n_0}(1 - \beta_i - 4i\frac{m_e}{m_i}\frac{\nu_e}{\omega})(1 - \frac{4}{3}i\frac{k_z^2 \eta_0^i}{\omega n_0 m_i}) = 0, \tag{6}$$

where

$$\beta_\alpha = \frac{2}{3}i\frac{k_z^2 \chi_\parallel^\alpha}{\omega n_0}, \quad \omega_e = \frac{k_y v_{Te}^2}{\Omega_e}\frac{\partial \ln n_0}{\partial x}.$$

1) Let $|\beta_i| \ll 1 \ll |\beta_e|$. Leaving out small imaginary terms and assuming that $\omega \gg (m_e/m_i)\nu_e$, we get

$$1 + \frac{\omega_e}{\omega} - \frac{k_z^2 v_{Ti}^2}{\omega^2}[\frac{8}{3} + \frac{\omega_e}{\omega}(\frac{2}{3} - \frac{\partial \ln T_0}{\partial \ln n_0})] = 0. \tag{7}$$

Consequently when $\omega < \omega_e$ we have

$$\omega^2 = k_z^2 v_{Ti}^2(\frac{2}{3} - \frac{\partial \ln T_0}{\partial \ln n_0}). \tag{8}$$

If $\partial \ln T_0/\partial \ln n_0 > 2/3$, then aperiodic instability sets in with an increment $\gamma \sim k_z v_{Ti}$; on the other hand if $\partial \ln T_0/\partial \ln n_0 < 2/3$, then the increments of the oscillations are determined by small dissipative terms, and it can be seen from Eq. (6) that the longitudinal motion leads to slight damping of the oscillations. In the case when $|\partial \ln T_0/\partial \ln n_0| \gg 1$ the instability can develop also in the region $\omega > \omega_e$. The frequency spectrum is then determined by the relation

$$\omega^3 = -k_z^2 v_{Ti}^2 \omega_e \frac{\partial \ln T_0}{\partial \ln n_0}. \tag{9}$$

This type of instability has an analog in a collisionless plasma [3] and is generally speaking very dangerous for a plasma.

In the region $\omega \approx (m_e/m_i)\nu_e$ the oscillations are damped. At lower frequencies ($\omega \ll (m_e/m_i)\nu_e$ and $\omega_e \gg \omega\beta_e$), instability can set in when

$$\omega = [8.9\frac{k_z^2 v_{Ti}^2}{\omega_e} - i(1 - \frac{3}{2}\frac{\partial \ln T_0}{\partial \ln n_0})(\frac{k_z^2 \chi_\parallel^e}{3Sn_0} - 4\frac{m_e}{m_i}\nu_e)](3.45 + \frac{3}{2}\frac{\partial \ln T_0}{\partial \ln n_0})^{-1} \ll k_z v_{Ti}.$$

However, it exists only in a narrow region of the plasma parameters.

2) Let now $|\beta_{e,i}| \ll 1$. If also $\omega \gtrsim (m_e/m_i)\nu_e$ (i.e., $\omega \gg k_z v_{Ti}$), then we get from (6)

$$\omega = \frac{k_y v_{Ti}^2}{\Omega_e} \frac{\partial}{\partial x} \ln n_0 T_0^{1.71} + \frac{2}{3} i(1 + S) \frac{k_z^2 \chi_\parallel^e}{n_0} \left(\frac{\partial \ln T_0}{\partial \ln n_0} + \frac{4}{3} \frac{k_z^2 v_{Ti}^2}{\omega_e^2} \right) \frac{\partial \ln n_0}{\partial \ln n_0 T_0^{1.71}}. \quad (10)$$

Instability is possible only in a plasma with inhomogeneous temperature. On the other hand, if $\omega \ll (m_e/m_i)\nu_e$ we get from (6)

$$1 + \frac{k_y v_{Te}^2}{\omega \Omega_e} \frac{\partial}{\partial x} \ln n_0 T_0^{(1+S)} - \frac{2}{3} \frac{k_z^2 v_{Ti}^2}{\omega^2} \left[5 - 2S \frac{\omega_e}{\omega} \left(1 - \frac{3}{2} \frac{\partial \ln T_0}{\partial \ln n_0} \right) \right] = 0. \quad (11)$$

Consequently, we obtain for oscillations with frequencies $\omega < \omega_e$

$$\omega^2 = -\frac{4}{3} S k_z^2 v_{Ti}^2 \frac{\partial \ln n_0 T_0^{-3/2}}{\partial \ln n_0 T_0^{1.71}}. \quad (12)$$

It is seen that the oscillations are unstable in the region

$$-\frac{1}{1 + S} < \frac{\partial \ln T_0}{\partial \ln n_0} < \frac{2}{3}.$$

When $T_0 = \text{const}$, the oscillation spectrum (12) coincides with that obtained by Moiseev [1].

3) Let us consider the case $T_{0e} \neq T_{0i}$, when we must stipulate $\omega \gg (m_e/m_i)\nu_e$. The dispersion equation has here the form

$$-\omega^2 \left(1 - \frac{4}{3} i \frac{k_z^2 \eta_0^i}{n_0 m_i \omega} \right) \left[1 + \frac{\omega_e}{\omega} \left(1 + \frac{1 + S}{1 - \beta_e} \frac{\partial \ln T_{0e}}{\partial \ln n_0} \right) \right] + k_z^2 v_{Ti}^2 \left\{ 1 + \frac{2}{3(1 - \beta_i)} \right.$$

$$\left. + \frac{\omega_e}{\omega} \left[\frac{1 + S}{1 - \beta_e} \left(\frac{\partial \ln T_{0e}}{\partial \ln n_0} - \frac{2}{3} \right) \left(1 + \frac{2}{3(1 - \beta_i)} \right) - \frac{1}{1 - \beta_i} \left(\frac{\partial \ln T_{0i}}{\partial \ln n_0} - \frac{2}{3} \right) \left(1 + \frac{2}{3} \frac{1 + S}{1 - \beta_e} \right) \right] \right\}$$

$$+ k_z^2 v_s^2 \left[1 + \frac{2}{3(1 - \beta_e)} + \frac{\omega_e}{\omega} \frac{S}{1 - \beta_e} \left(\frac{\partial \ln T_{0e}}{\partial \ln n_0} - \frac{2}{3} \right) \right] = 0. \quad (13)$$

If $|\beta_i| \ll 1 \ll |\beta_e|$, then (13) reduces to

$$1 + \frac{\omega_e}{\omega} - \frac{k_z^2 v_{Ti}^2}{\omega^2} \left[\frac{5}{3} + \frac{T_e}{T_i} + \frac{\omega_e}{\omega} \left(\frac{2}{3} - \frac{\partial \ln T_{0i}}{\partial \ln n_0} \right) \right] = 0. \quad (14)$$

This equation is analogous to Eq. (7). It is easily seen that the expressions obtained above for the spectra of the oscillations (8) and (9) remain in force if it is recalled that the frequency ω_e contains the electron temperature. In the case when $|\beta_{e,i}| \ll 1$ the frequencies and the increments of the oscillations coincide in order of magnitude with those given by formula (10), if T_0 is replaced by T_{0e}.

We note in conclusion that the obtained instabilities lie in the frequency region $\sim k_z v_{Ti}$ both for isothermal and nonisothermal plasma. Such instabilities, being of long-wave type, can lead to anomalously large diffusion with a coefficient of the order of magnitude of that for Bohm diffusion. [1]

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1) The contribution of the lower-order terms, connected with motion transverse to the magnetic field, are of no significance in our analysis.

Errata

Article by I. S. Baikov in Vol. 4, No. 8.

On page 201, line 16 from top, substitute "will be paid" for "was paid." In line 20 from top substitute "We shall investigate" for "We investigated." In line 22 from the top substitute "hydrodynamics [1)]" for "hydrodynamics." In the first line of Eq. (1) substitute $\frac{\partial n_0}{\partial x}$ for $\frac{\partial \ln n_0}{\partial x}$. In the last line substitute "omitted" for "omitted [1)]."

On page 304 add the following sentence to the footnote: "Therefore the coefficients a^e an a^i should be set equal to zero."