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DOUBLY LOGARITHMIC ASYMPTOTIC EXPRESSION IN QUANTUM ELECTRODYNAMICS

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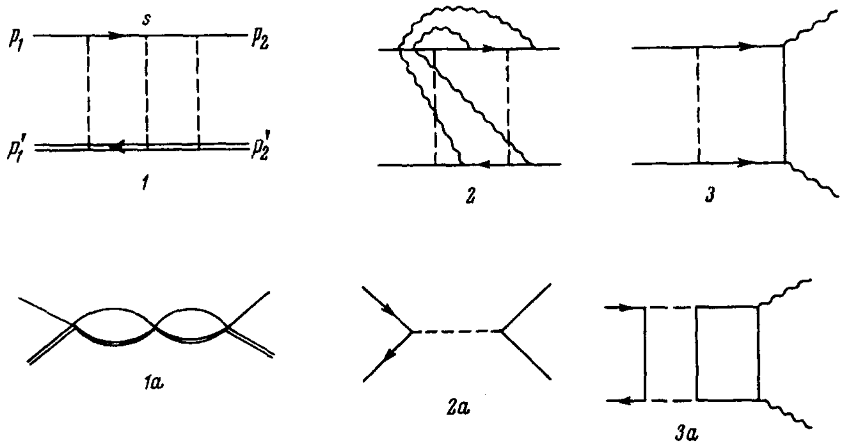
An asymptotic expression for a scalar four-point diagram at high energy $s = (p_1 + p_2)^2$ and at finite $t = (p_1 - p_1')^2$ in the case of an interaction of the $e\varphi^3$ type with $e^4 \ln s \ll 1$ was obtained by Polkinghorne [1]. This asymptotic expression is determined by a sequence of ladder diagrams 1 and is of the form

$$\sum_{n=1}^{\infty} \frac{\alpha}{s}(\alpha \ln s\ K(t))^{n-1}\frac{1}{(n-1)!} = \frac{\alpha}{s}\exp(\alpha K(t)\ \ln s); \quad \alpha = \frac{e^2}{4\pi} = 1/137. \tag{1}$$

An asymptotic expression for these diagrams is best obtained by the method of Sudakov [2], by resolving the intermediate integration momentum k into a longitudinal and transverse part, $k = u\vec{p}_1 + v\vec{p}_2 + \vec{k}_\perp$. The separation of ln s takes place in the integration with respect to u and v, and K(t) is a two-dimensional integral corresponding to one loop (Fig. 1a), constructed in accordance with the usual Feynman rules, with replacement of all the intermediate momenta k by $k_\perp$ and of d^4k by $d^2k_\perp$.

In the scalar case at a finite particle mass, the integrals corresponding to K(t) converge. Formula (1) is then the correct asymptotic expression, called singly-logarithmic, since for each power of α there is an equal power of ln s. If one of the particles has zero mass (photon), each loop (Fig. 1a) diverges logarithmically at small $k_\perp$.

If both particles have spin 1/2, then a logarithmic divergence occurs also for large $k_\perp$. Since the initial diagram (Fig. 1) shows now divergence in either case, the asymptotic expression (1) is no longer valid. Actually the integration with respect to $k_\perp$ in the loop is cut off at values of the order of s^{-1} for small $k_\perp$ and of the order of s for large $k_\perp$. Therefore the arising logarithmic instability corresponds to the appearance of an additional ln s for each loop. In this case we already have $\ln^2 s$ for each power of α, and the asymptotic expression is therefor called doubly logarithmic. The doubly-logarithmic terms con-



nected with divergences of the photon lines at small $k_\perp$ are always a result of integration over the region corresponding to real photons: $k^2 = k_\perp^2 + k_\parallel^2 = 0$. Such intermediate photons are similar to real bremsstrahlung photons which, as is well known, also lead to double logarithms in the cross sections; we shall therefore call these photons intermediate bremsstrahlung photons. These photons are denoted in the figure by wave lines to distinguish them from other photons (dashed).

Let us consider the possible doubly-logarithmic asymptotic expressions for four-point diagrams in quantum electrodynamics. It is convenient to classify all processes with respect to the charge Z propagating in the intermediate state of the t-channel.

When Z = 0 there are two fermions and two photons in the initial and final states, with the arrows of the fermion lines oppositely directed. Then there is either no charge at all (scattering of light by light), or the charge moves in the initial direction at the end of the process. Such processes include: forward annihilation e^+e^- into $\mu^+\mu^-$ (I) [1], forward e^+e^- and e^-e^- scattering (II), and forward Compton scattering (III). Since the direction of charge motion does not change, we should expect the absence of a doubly-logarithmic contribution from bremsstrahlung quanta. Actually all the doubly-logarithmic contributions of the intermediate bremsstrahlung photons shown in Fig. 2 are cancelled out among the different diagrams. In this case the doubly-logarithmic contribution is determined only by diagrams containing divergences at large $k_\perp$ from two fermion lines (1a), which, as in the scalar case, form the ladder sequence 1. The result of its summation has the same form for all the processes:

$$A(\xi) = f_0 \frac{2}{\xi} I_1(\xi); \quad \xi^2 = \frac{2\alpha}{\pi} \ln^2 s, \tag{2}$$

where f_0 is the Born amplitude of the process and $I_1(\xi)$ is a Bessel function of imaginary argument. The asymptotic expression (2) corresponds to the presence of branching in the partial wave of the t-channel, of the form $(e^2 - 4\alpha/\pi)^{1/2}$. A similar asymptotic expression was observed first in the model case by Bjorken and Wu [3].

Process I has only a doubly-logarithmic asymptotic expression, whereas processes II and III, with identical fermions, are characterized also by diagrams of the type of Figs. 2a and 3a, with photons in the intermediate state. The contributions from these processes are proportional to s and are principal. However, owing to the presence of an additional factor α in the amplitudes shown in Fig. 3a, it turns out to be comparable with the doubly-logarithmic asymptotic expression for the process III when $\alpha(\ln^2 s)/\pi \sim 1$. The same holds for the scattering of light by light. Doubly-logarithmic asymptotic expressions for the processes I, II, and III were considered earlier by Abrikosov [4] and by Milekhin and Fradkin [5], but they obtained incorrect results.

When Z = 1, one of the initial lines in diagram 1 should be a photon line. The doubly-logarithmic asymptotic expression arises in this case only as a result of bremsstrahlung quanta; it has been considered in [6].

When Z = 2, all initial and final particles are charged, and the direction of motion of both charges is reversed. In this case the doubly-logarithmic contribution is due both to large $k_\perp$ and to bremsstrahlung photons. Such processes are backward e^-e^+ and $e^-\mu^+$ scattering and backward annihilation of an e^-e^+ pair into a $\mu^-\mu^+$ pair. The essential doubly-logarithmic diagrams of these processes have the form shown in Fig. 2, under the condition that the fermion lines have the same direction. The contributions of bremsstrahlung photons, which cancel out when Z = 0, now are additive. There are no diagrams of the type 2a. We have found that the doubly-logarithmic asymptotic expression for the amplitude of these processes is

$$A(\xi) = \exp(-\xi^2/2) f_0 \frac{2i}{\pi} \int_{C-i\infty}^{C+i\infty} dl \, \exp(l\xi) \frac{l}{dl} \ln D_{-\frac{1}{4}}(l), \tag{3}$$

where

$$\xi^2 = \frac{2\alpha}{\pi} \ln^2 s.$$

$D_{-\frac{1}{4}}(l)$ is a parabolic-cylinder function. The exponential factor ahead of the integral in (3) is due to the contribution of the bremsstrahlung photons with $|k_\perp|^2 \ll 1$. For large ξ, the asymptotic expression is determined by the extreme right pair of complex-conjugate zeroes of $D_{-\frac{1}{4}}(\xi)$ and the amplitude oscillates:

$$A(\xi) = \begin{cases} -8e^{-2.261\xi} \cos 1.843\xi + \ldots & \xi \gg 1 \\ 1 - \frac{5}{8}\xi^2 + \frac{35}{192}\xi^4 + \ldots & \xi \ll 1. \end{cases} \tag{4}$$

The cross section for the processes for Z = 0 is given by the square of formula (2). When Z = 2, the total cross section for elastic scattering and bremsstrahlung of real quanta with $|k_\perp|^2 \ll 1$ is equal to the square of (3) without the exponential factor ahead of the integral.

These results, together with the investigation of doubly-logarithmic asymptotic expressions of various processes at large s, t, and u [7-8], cover all the doubly-logarithmic expressions for two-particle processes in quantum electrodynamics.

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CONSISTENT RELATIVIZATION OF AN SU(6) GROUP FOR TWO-PARTICLE REACTIONS

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1. So far, no one has succeeded in finding a relativistic version of the SU(6) group with internal consistency, i.e., one leaving invariant the free equations and leading to noncontradictory limitations on the reaction amplitudes. The only success in this direction was attained for collinear processes, for which the group $SU_W(6)$ was found [1].

We propose here a group $SU_x(6)$, isomorphic to SU(6), satisfying the foregoing requirements, and applicable to two-particle reactions without being confined to collinearity 1).

2. Let us examine this group with quarks as the example. For quarks, the corresponding transformations are written in the form

$$\delta\Psi(p) = [\frac{i}{2}\omega^a\lambda_a + e^i_\mu(p)\gamma_\mu\gamma_5(\alpha_i + \alpha^a_i\lambda_a)]\Psi(p), \tag{1}$$

where $e^i_\mu(p)$ are three vectors that are orthogonal to one another and to the momentum p: $(e^i\cdot p) = 0$, $(e^i\cdot e^j) = \delta_{ij}$, λ_a are 8 Gell-Mann matrices, and ω^a, α_i, and α^a_i are the transformation parameters. These transformations commute with the free Dirac equation. The vectors $e^i_\mu(p)$ can be expressed in many ways in terms of the momenta of the particles that participate in the reactions. For two-particle reactions, we think that the physically most natural choice of basis is:

$$e^1_\mu(p) = N_1(P_\mu - \frac{p^2}{(Pp)}p_\mu),\ e^2 = N_2\epsilon_{\mu\nu\lambda\rho}p_{1\nu}p_{2\lambda}p_{3\rho},\ e^3_\mu(p) = N_3\epsilon_{\rho\nu\lambda\rho}e^1_\nu e^2_\lambda p_\rho, \tag{2}$$