Fig. 3. Other transverse momenta (shown dashed in Fig. 3) were calculated relative to the center of gravity of the secondary particles, using statistical weights proportional to the corresponding energies. As seen from the figure, these transverse momenta are relatively small.

It seems to us that all the transverse momenta obtained in this manner can be regarded as realistic. Of course, they can be influenced by all the uncertainties connected with the errors in the determination of the primary energy and of the angular distribution, but this cannot change the overall picture of the transverse-momentum distribution. The average value of $p_\perp$ is $\langle p_\perp \rangle \approx 1 - 2$ GeV/c.

* A complete analysis of the interactions is described in the preprints of the Institute and has been submitted to Nuovo Cimento.

COLLECTIVE COULOMB EXCITATION OF NUCLEI IN A REGULAR CRYSTAL

Yu. M. Kagan and F. N. Chukhovskii
Submitted 8 December 1966
ZhETF Pis'ma 5, No. 5, 166-170, 1 March 1967

1. It has been shown that in resonant interaction of γ quanta or particles with nuclei situated in a crystal lattice, the resultant excited state (compound nucleus) has a collective character. In other words, the excited state is in this case not an excitation of an individual nucleus, but is "smeared" over the entire crystal.

It turns out that such a collective state can occur under certain conditions following Coulomb excitation of low-lying levels of nuclei which accompanies the scattering of fast charged particles in a regular crystal. When such an excited state decays, the angular distribution of the γ quanta has a unique character which differs greatly from the usual case.

2. Let the Coulomb excitation be realized by a beam of fast heavy particles (for concreteness, protons) with initial momentum p. As a result of the interaction, the nuclei of the crystal acquire a momentum q = p - p', where p' is the final momentum of the particles.

In an ideally rigid lattice, the γ radiation of the n-th nucleus will have a phase $\exp[i(q - \kappa)R_n]$ (κ - wave vector of the γ quantum). In a real crystal, such a phase is maintained only if no phonon excitation takes place, i.e., the process will have a "recoilless" character.

The phononless-transition probability depends on the setup of the experiment. We can visualize two types of experiments: a) an isomer level of the Mossbauer type is excited and only the γ quanta for which the Mossbauer effect takes place are registered; b) all the γ quanta are registered, regardless of their energy. A suitable analysis shows that in this case

$$a)\ f(q,\kappa) = \exp[-Z(q) - Z(\kappa)],\quad b)\ f(q,\kappa) = \exp[-Z(q-\kappa)], \tag{1}$$

where Z is the usual exponent in the Debye-Waller factor. If the state of the lattice does not change, then the Coulomb excitation and the subsequent γ decay have a coherent character. The matrix element for such a transition is written in the form

$$M_{\text{coh}} = \sum_n M_0(q, \kappa/\kappa)\,[f(q, \kappa]^{1/2} \exp[i(q-\kappa)R_n] \tag{2}$$

When the protons are scattered to angles of some magnitude, the momentum transfer turns out to be so large that f, together with (2), practically vanishes. The coherent amplitude turns out to be different from zero only at small scattering angles θ. In this angle region we have (u is the velocity of the incident particles)

$$q^2 = q^2_{\min} + p^2\theta^2, \quad q_{\min} = \kappa\frac{c}{u} \tag{3}$$

and for real values of κ one can speak of angles $\theta \approx q_{\min}/p$. By virtue of this, for transitions corresponding to a multipolarity E2 and higher, the coherent part of the total cross section will be at least $(q_{\min}/p)^2$ times smaller than the ordinary cross section, and in practice it cannot be determined experimentally.

However, for multipolarities E1 and M1 an appreciable part of the cross section is concentrated precisely in the small region of proton scattering angles $\approx q_{\min}/p$. This makes possible coherent processes for such transitions by Coulomb excitation in the crystal.

3. The differential cross section for coherent emission of γ quanta can be represented in the form

$$d\sigma_{\text{coh}} = g\sum_K |M_0(q,\frac{\kappa}{\kappa})|^2 f(q, \kappa)\frac{1}{v_0}\delta(q-\kappa+K)\delta(\epsilon-\epsilon'-\omega_0)\,d^3p'\,d\Omega_\kappa. \tag{4}$$

Here v_0 is the volume of the unit cell, K the reciprocal-lattice vector multiplied by 2π, g is a constant, and $\omega_0 = \kappa c$. We integrate over a finite proton momentum. This eliminates the momentum δ function, and the argument in the energy δ function now vanishes under the condition

$$\cos(\hat{\kappa p}) = \frac{c}{u} - p\frac{K}{p\kappa} - \frac{(\kappa-K)^2}{2p\kappa}. \tag{5}$$

In the scattering-angle interval of interest to us, the last term of (5) is small and can be neglected. We can then conclude from (5) that the γ quanta will be emitted in cones with axes along p. Let the crystal have cubic symmetry and let the protons move along the cubic axis. Then

$$\cos(\hat{\kappa p}) = \frac{c}{u} - \frac{K_x}{\kappa}.$$

The maximum number of emission cones is obviously simply equal to

$$1 + [\frac{2\kappa}{K^0_x}],$$

where K_x^0 is the reciprocal-lattice basis vector, and [...] denotes the integer part. If we consider excitation of low-lying levels only (of the isomer type), then the number of such cones is quite limited.

The total cross section for an individual cone (a) can be written, with (4) and (5) taken into account, in the form

$$\sigma^a_{\text{coh}} = g \sum_{K_\perp} |M_0(q, \frac{\kappa}{\kappa})|^2 f(q, \kappa_a) \frac{2\pi m}{p\kappa v_0}|_{q=\kappa_a + K;\; K=K^a_x + K_\perp}. \tag{6}$$

The summation here is over the reciprocal-lattice vectors lying in the plane perpendicular to the x axis.

4. For E1 and M1 we have ($\theta \ll 1$) [4]

$$|M_0^{(E1)}(q, \frac{\kappa}{\kappa})|^2 = [1 - 2A_2^{(E1)} P_2(\frac{q}{q}) P_2(\frac{\kappa}{\kappa})] \frac{p^2}{q^2};$$

$$|M_0^{(M1)}(q, \frac{\kappa}{\kappa})|^2 = [1 + A_2^{(M1)} P_2(\frac{\kappa}{\kappa})] p^2 \frac{(q^2 - q^2_{\min})}{q^4}. \tag{7}$$

A large number of terms in the sum over the reciprocal-lattice points contribute to the cross section (6). This makes it possible to change from summation to integration, making the substitution

$$\sum_{K_\perp} \to \frac{s_0}{(2\pi)^2} \int d^2 K_\perp.$$

Substituting (7) in (6) and extending the integration to ∞ (which can always be done if account is taken of the exponential character of the dependence in f), we obtain for the case when the cone angle is small

$$\sigma^a_{\text{coh}} = g \frac{mp}{2(\kappa a)} f(q_{\min}) e^{Z(q_{\min})} \begin{cases} -Ei[-Z(q_{\min})][1 + A_2^{(E1)} + 3A_2^{(E1)} Z(q_{\min})] - \\ \quad -3A_2^{(E1)} e^{-Z(q_{\min})} \\ \\ (1 + A_2^{(M1)})[-Ei(-Z(q_{\min}))(1 + Z(q_{\min})) - e^{-Z(q_{\min})}]. \end{cases}$$
$$v_0 = s_0 a \tag{8}$$

This is the final expression for the cross section, corresponding to each individual cone, for coherent emission of γ quanta in the case of Coulomb excitation of the nuclei.

It is of interest to compare (8) with the usual total cross section σ_0 for an individual nucleus. It is easy to show that (we confine ourselves to the E1 case)

$$\frac{\sigma^a_{\text{coh}}}{\sigma_0} \approx \frac{\pi}{2(\kappa a)} \frac{\{-Ei[-Z(q_{\min})]\}}{\ln(2p/q_{\min})}. \tag{9}$$

We see therefore that to obtain a noticeable coherent effect it is necessary: 1) to excite the lowest-lying levels of the isomer type, 2) to use beams of sufficiently fast particles so that q_{min} does not differ strongly from κ.

5. When the primary particle moves in a crystal, it looses energy. This imposes limitations on the crystal thickness within which coherence is maintained. The corresponding value is obtained from the condition that the phase will not change over the thickness by more than π:

$$\frac{q_{min} l \Delta\epsilon}{2\epsilon} \approx \pi . \tag{10}$$

We note that this condition also calls for high-energy particles.

6. We remark in conclusion that, in principle, a process which in some sense is the inverse of that considered above can exist, namely a specific Coulomb excitation of an individual nucleus by the periodic field of a crystal lattice. This phenomenon, to which attention was first called by V. V. Okorokov [5], calls for a special study.

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τ DECAY AND CURRENT ALGEBRA

A. I. Vainshtein, A. D. Dolgov, V. I. Zakharov, and A. B. Kaidalov

Submitted 12 December 1966
ZhETF Pis'ma 5, No. 5, 170-173, 1 March 1967

1. Nonleptonic decays of K mesons were considered in a number of papers [1-4] within the framework of the hypothesis of partially conserved axial current (PCAC) and current algebra. Assuming that the weak-interaction Hamiltonian H is in the form of a product of a current by a current, and that the matrix elements vary slowly when the 4-momenta of the pions approach zero symmetrically, Suzuki [1] proved the $\Delta = 1/2$ rule and obtained a relation between the probabilities of the $K \to 3\pi$ and $K \to 2\pi$ decays. However, inasmuch as in the limit of zero pion momenta the amplitude depends on the method of going to the limit, the assumed slow variation is not justified. To take into account a fast variation, it was proposed in [3] to expand the amplitudes in the pion energies, but the cause of the rapid variation was not discussed.

In the case of the $K \to 2\pi$ decay, as noted in [4], the ambiguity in the calculation of the limit of the amplitude can be explained by means of a pole diagram (Fig. 1).

2. In this note we consider the consequences of the assumption that the $K \to 3\pi$ decay amplitude is a constant plus a rapidly-varying contribution from the pole diagrams shown in