Finally, when $\lambda \gg B \gg x_0$ the principal role is assumed by the electric field, and the voltage-current characteristic should have the same form as in the absence of the sound flux. Thus, the general appearance of the voltage-current characteristic should be such as shown in the figure.

The presence of a descending section on the voltage-current characteristic can lead to development of domain instability [5,6].

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COLLECTIVE PROPERTIES OF LARGE-RADIUS EXCITONS

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In certain semiconductors (Ge, CdS. GaAs) the Bohr radius of the exciton becomes quite large, $a_0 = \hbar^2\epsilon/me_0^2 \sim 10^{-6}$ cm. Excitons with such a radius begin to "touch" at relatively low densities $n \sim a_0^{-3} \sim 10^{18}$ cm^{-3}, which are perfectly attainable by modern experimental means. It is clear that when $na_0^3 \sim 1$ the excitons influence one another greatly and, in particular, the Bohr level of the exciton $E_0 = -me_0^4/2\hbar^2\epsilon^2 \sim -10^{-2}$ eV should, in general, change by an amount equal to its own value. It is obvious that when $na_0^3 \gtrsim 1$ the exciton is not a good quasiparticle [1] and the model of non-ideal Bose gas of excitons is just as poor as the model of a non-ideal Bose gas of Cooper pairs for a superconductor. The limiting case $na_0^3 \gg 1$ was considered earlier [2,3]. In this article we report results pertaining to the opposite case, $na_0^3 \ll 1$. We shall show that in this limit the excitons cannot be regarded rigorously as Bose particles when speaking of collective effects. The reason is that the properties of the electrons and holes forming the excitons become renormalized as functions of n. Allowance for this fact introduces in all the formulas contributions of the same order as allowance for the non-ideal nature of the exciton gas. A rigorous investigation of this question is especially interesting in connection with the fact that many workers [4-6] have predicted Bose-condensation of excitons on the basis of the concept of excitons as Bose particles.

We consider a two-band semiconductor model with dispersion law $E_c(\vec{p}) = E_g + \vec{p}^2/2m_c - \mu_c$ and $E_v(\vec{p}) = -\vec{p}^2/2m_v - \mu_v$, where μ_c is the chemical potential in the conduction band and μ_v in the valence band. The electrons interact in accord with Coulomb's law with effective charge $e = e_0/\sqrt{\epsilon}$, so that

$$H_{int} = \frac{1}{2}\sum_{\vec{p},\vec{p}',\vec{q}} V_{\vec{q}}\,(a^+_{c\vec{p}}\, a^+_{c\vec{p}'}\, a_{c\vec{p}'+\vec{q}}\, a_{c\vec{p}-\vec{q}} + a^+_{v\vec{p}}\, a^+_{v\vec{p}'}\, a_{v\vec{p}'+\vec{q}}\, a_{v\vec{p}-\vec{q}} + 2\, a^+_{c\vec{p}}\, a^+_{v\vec{p}'}\, a_{v\vec{p}'+\vec{q}}\, a_{c\vec{p}-\vec{q}}); \qquad V_{\vec{q}} = 4\pi e^2/\vec{q}^2 \tag{1}$$

The lifetime of the exciton is large compared with all the relaxation times, so that the system of excitons can be regarded as being in thermodynamic equilibrium at a specified concentration. We put $T = 0$. In the presence of coupled electron-hole pairs (excitons), the correlators $\langle a_{v\vec{p}} a^+_{c\vec{p}} \rangle$ and $\langle a_{c\vec{p}} a^+_{v\vec{p}} \rangle$ have nonzero values. Corresponding to these correlators are the Green's functions $G_{vc}(p)$ and $G_{cv}(p)$, which we introduce in addition to the ordinary functions $G_c(p)$ and $G_v(p)$. The set of Green's functions satisfies the system of Dyson's equations

$$\begin{cases} G_c^{(0)-1}(p)\, G_c(p) = 1 + \Sigma_c(p)\, G_c(p) + \Sigma_{cv}(p)\, G_{vc}(p)\,, \\ G_v^{(0)-1}(p)\, G_{vc}(p) = \Sigma_{vc}(p)\, G_c(p) + \Sigma_v(p)\, G_{vc}(p)\,. \end{cases} \tag{2}$$

This system is similar to the Gor'kov equations. We shall solve it first in the first approximation with respect to the interaction. For the self-energy part $\Sigma_{vc}(p)$ we obtain the equation

$$\Sigma_{vc}(\vec{p}) = \text{v} \frown \text{c} = \int \frac{d^3p'}{(2\pi)^3}\, V_{\vec{p}-\vec{p}'}\, \frac{\Sigma_{vc}(\vec{p}')}{2\epsilon(\vec{p}')}\,, \tag{3}$$

where

$$\epsilon^2_{\vec{p}} = E^2_{\vec{p}} + \Sigma^2_{vc}(p)\,, \quad E_{\vec{p}} = \frac{\vec{p}^2}{2m} - \frac{\mu_c - \mu_v}{2} - \frac{\Sigma_c(\vec{p}) - \Sigma_v(\vec{p})}{2}\,,$$

$$\frac{2}{m} = \frac{1}{m_c} + \frac{1}{m_v}\,.$$

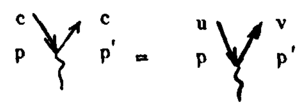
Equation (3) is similar to the equation for Δ in superconductivity theory. Let us expand $\epsilon_{\vec{p}} = E_{\vec{p}} + \Sigma^2_{vc}(\vec{p})/2\epsilon_{\vec{p}}$, regarding $\Sigma_{vc}(\vec{p})$ as a small quantity. Then (3) takes the form of a Schrödinger equation with respect to $v_{\vec{p}} = \Sigma_{vc}(p)/2\epsilon_{\vec{p}}$, with a Coulomb potential plus a small perturbation that depends on $\Sigma_{vc}(\vec{p})$ (or on $v_{\vec{p}}$). We obtain the normalization conditions for $v_{\vec{p}}$ by expressing the exciton density in terms of the solution of the system (2):

$$n = \int \frac{d^3p}{(2\pi)^3}\, \frac{\epsilon_{\vec{p}} - E_{\vec{p}}}{2\epsilon_{\vec{p}}} = \int \frac{d^3p}{(2\pi)^3}\, V^2_{\vec{p}}\,. \tag{4}$$

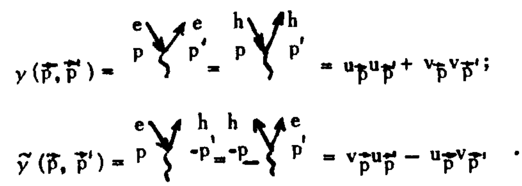
The exciton level (the chemical potential of the exciton) $\mu_e = \mu_c - \mu_v$ can be determined by finding the eigenvalue of (3). We shall assume henceforth that $e = a_0 = m/2 = 1$. In the zeroth approximation $\mu_e = E_0 = -1/2$, and $v_{\vec{p}}$ is the wave function of the ground state in the Coulomb field, normalized in accord with condition (4): $v_p = 8\sqrt{\pi n}/(\vec{p}^2 + 1)^2$. The exciton level with the correction is equal to $\mu_e = -1/2 + 13\pi n/3$. In this approximation (first order in $V_{\vec q}$), the Green's functions are

$$G_c(p) = u_{\vec p}^2 G_e(p) - v_{\vec p}^2 G_h(-p), \quad G_{ve}(p) = u_{\vec p} v_{\vec p} [G_e(p) + G_h(-p)] ;$$
$$G_e(p) = \left(\omega - \frac{m_v - m_c}{m_v + m_c}\,\frac{\vec p^2}{2m} - \epsilon_p + i\delta\right)^{-1}, \; G_h(p) = \left(\omega + \frac{m_v - m_c}{m_v + m_c}\,\frac{\vec p^2}{2m} - \epsilon_{\vec p} + i\delta\right)^{-1}; \quad u_{\vec p}^2 = 1 - u_{\vec p}^2. \tag{5}$$

The result corresponds to the canonical tranformation $a_{c\vec p} = u_{\vec p} a_{e\vec p} - v_{\vec p} a^{+}_{h,-\vec p}$ and $a_{v\vec p} = v_{\vec p} a_{e\vec p} + u_{\vec p} a^{+}_{h,-\vec p}$ to new Fermi operators (we have simultaneously gone over to the hole representation in the valence band). In going to the new representation, the matrix elements of the interaction change in the following fashion: instead of a single vertex of the type



there appear vertices of two types



The correction $\Delta\mu_e^{(1)} = 13\pi n/3$ for μ_e was found in first order in $V_{\vec q}$. It can be shown that diagrams more complicated than (3) for the self-energy parts Σ_c, Σ_v, and Σ_{vc} also yield corrections that are linear in n for μ_e. The summation of the entire infinite sequence of such diagrams yields the contribution $\Delta\mu_e^{(2)}$ shown schematically in Fig. 1.

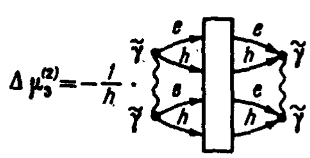


Fig. 1

The "brick" in this figure denotes that four Fermi lines are interconnected in all possible manners by interaction lines (the brick does not contain the vertices $\tilde\gamma \sim \sqrt{\tilde n}$). The Fermi lines are shorted on the left by the amplitude of creation of two particle pairs, and on the right by the annihilation amplitude. Entering in the "brick" are the scattering amplitudes of two, three, and four particles by one another in vacuum, and also a term in which all four lines are free.

To determine whether the ground state is stable (it is Bose-condensed, since $G_{vc}(p) \neq 0$)

we must investigate the spectrum of the collective excitations. We consider to this end the equations for the two-particle Green's functions (see Fig. 2). The terms are grouped in these equations in such a way that the shaded blocks with four ends represent the aggregate of diagrams which do not break up into two parts. U and $\tilde{U}$ denote vertices that are irreducible in

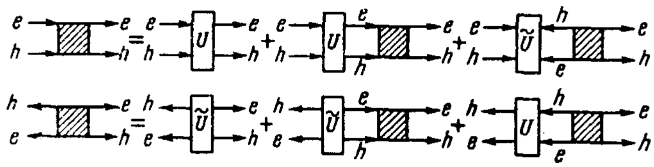


Fig. 2

terms of the lines e and h having the same direction.

The equations shown schematically in Fig. 1 describe the properties of a particle-hole pair. They are similar in form to the Belyaev equations [7] for the Green's functions of a non-ideal Bose gas, but, unlike the latter, they are integral. Just as Eq. (3) for $\Sigma_{vc}(p)$, they are weakly-perturbed Schroedinger equations with a Coulomb potential, and can be solved in first order in n. The pole of the corresponding solution determines the acoustic dispersion law $\omega(k) = [s^2\vec{k}^2 + (\vec{k}^2/2M)^2]^{1/2}$, where $s = \sqrt{\mu_e/M}$ and $M = m_c + m_v$. This result, according to which $\omega(0) = 0$, is based on a certain relation between the self-energy and vertex parts of the diagram, which results from the fact that the contribution from the higher orders in U and $\tilde{U}$ (starting from the second) is expressed by the same eight-point block as $\Lambda\mu_e^2$. By way of an example, Fig. 3 shows the vertex $U^{(2)}$.

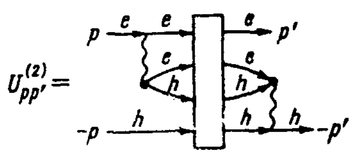


Fig. 3

Thus, a low-density exciton system has many properties of a weakly non-ideal Bose gas and, in particular, is superfluid. However, there is a significant difference from a Bose gas, consisting in the fact that μ_e and s are expressed not only in terms of the amplitude of scattering of two excitons by each other, but contain also an essentially positive increment connected with the renormalization of the properties of the electrons and the holes and of the interaction between them. As a result, the system can be stable even in the presence of weak attraction between the excitons.

A formula similar (6) for a system of fermions of the same sort, with an interaction that should depend essentially on the spins, was obtained by V. N. Popov [8].

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SCATTERING OF LIGHT BY COHERENT SPIN WAVES

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A study of the scattering of light in magnetically-ordered crystals yields important information on the dynamics of a spin system. Raman scattering of light was recently observed [1] in antiferromagnetic FeF_2. This effect, however, is quite weak, owing greatly to the smallness of the amplitude of the thermal spin waves.

We consider in this letter Raman scattering of light by coherent spin waves excited, for example, in parametric fashion. An important fact is that in such an excitation method the experimentally attainable energy of the monochromatic spin waves exceeds kT by 15 - 16 orders of magnitude [2].

In addition, the length of such spin waves can be varied in a wide range ($0 < k_m < 10^5$ cm^{-1}) with the aid of a magnetic field, and is comparable with the wavelength of light. All this makes it possible to observe intense scattering of light at any angle ($0 - \pi$), whose magnitude depends on the external magnetic field.

Let us consider for simplicity a standing spin wave

$$\vec{m}(\vec{r}, t) = \vec{m} \cos(\vec{k}_m \vec{r}) \cos \Omega t, \tag{1}$$

where $\vec{k}_m \perp \vec{H}_0$ and $\Omega = \Omega_H/2$. This corresponds to the case of "parallel pumping" with frequency Ω_H. Let light with frequency $\omega \gg \Omega$ and wave vector $\vec{k}_p$ be incident on a ferromagnet in which the wave (1) is excited. We shall assume that either of the two Raman-scattering mechanisms - the "magnetic" or "electric," proposed respectively by Bass and Kaganov [3] and by Elliott and London [4] - predominates. Then the differential scattering cross section, obtained by one of the authors [6] takes the form

$$\frac{d\sigma}{d\theta} = \frac{\Phi^2 k_p^2}{(4\pi)^2} \sum_{j=1}^{3} \left| \int d^3r \, \frac{m_j(r) e^{-iqr}}{M} \right|^2 . \tag{2}$$

Here Φ is the angle of rotation of the polarization plane per unit length of sample when the light propagates along the magnetization $\vec{M}$ (Faraday effect), $\vec{q} = \vec{k}_p - \vec{k}'_p$, $\vec{k}'_p$ is the wave