mean free path. On the other hand, if the state is currentless, then an increase in the mean free path leads to a decrease in the corresponding component of the electric conductivity (a diffusion situation is realized on the basis of new states, see the derivation of formula (3)).

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## THERMAL PROPERTIES OF ANOMALOUS SUPERCONDUCTORS

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It is well known that the experimental data for different properties of the so-called anomalous superconductors (which include primarily Pb, Hg, Nb, and NbN) are in poor agreement with the theoretical formulas obtained in ordinary superconductivity theory (see [1]). In these superconductors, the electron-phonon interaction is not weak and consequently the ratio $\pi T_k/\theta$ ($\theta$ - Debye temperature) is not negligibly small (for example, $\pi T_k/\theta \simeq 0.25$ for Pb).

The ratio $\Delta(o)/T_k$ and other characteristics of anomalous superconductors were calculated in [2] numerically and in [3] analytically on the basis of the Froehlich model, which takes direct account of the interaction of the electrons with the lattice.

We consider here the jump of the specific heat on going from the superconducting to the normal state, and the behavior of the thermal conductivity near $T_k$ for superconductors with strong coupling.

We write the equation for the self-energy part $\Sigma(\omega_n, T)$, describing the pairing of the electrons [4]:

$$\Sigma(\omega_n, T) = \frac{T}{(2\pi)^3} g^2 \sum_{\omega_{n'}} \int d\mathbf{k} \frac{\omega^2}{\omega^2 + (\omega_n - \omega_{n'})^2} \cdot \frac{\Sigma(\omega_{n'}, T)}{\omega_{n'}^2(1+\gamma\Sigma^2/\omega^2)+\xi^2+\Sigma^2(\omega_{n'}, T)}, \quad (1)$$

$\omega_n = (2n + 1)\pi T$, $\omega$ - phonon energy. The term $\sim\omega_{n'}^2\Sigma^2/\omega^2$ in the denominator of the intergrand (1) is the result of the $\Sigma$-dependence of the function $\Sigma_1(\omega_n, T)$ which describes the scattering (we shall not write out the corresponding expression in detail). By regarding the addition to $\Sigma_1$, due to $\Sigma$, as a perturbation we obtain the term $\sim\omega_n^{2\prime}\Sigma^2/\omega^2$. The scattering processes lead essentially to a renormalization of the chemical potential and to an attenuation which is significant only in the direct vicinity of $T_k$ and when $\omega_n' \sim \theta$ [4]. We seek the solution of (1) in the form

$$\Sigma(\omega_n, T) = \Sigma_0 + \Sigma'; \quad \Sigma_0 = C(T)\,\frac{\omega^2}{\omega^2 + \omega_n^2}. \tag{2}$$

$\Sigma' \ll \Sigma_0$, and the function $C(T) \equiv \Sigma(o, T)$ is calculated with the aid of (1).

To find the sought function $C(T)$ as $T \to T_k$, we investigate the following equation, which follows from (1):

$$T\sum_{\omega_{n'}}\int dk\,\frac{\omega^2}{\omega^2+\omega_{n'}^2}\cdot\frac{\Sigma(\omega_{n'},T)/C}{\omega_{n'}^2+\xi^2}\Big|_{T=T_k} = T\sum_{\omega_{n'}}\int dk\,\frac{\omega^2}{\omega^2+\omega_{n'}^2}\times$$
$$\times\frac{\Sigma(\omega_{n'},T)/C}{\omega_{n'}^2(1+\gamma\Sigma^2/\omega^2)+\xi^2+\Sigma^2(\omega_{n'},T)}\Big|_{T\to T_k}. \tag{3}$$

Near $T_k$ we have $\Sigma/\omega_n \ll 1$. We carry out the appropriate expansion in the right side of (3) and solve the resultant equation (the function $\Sigma' = \Sigma(\omega_n, T) - \Sigma_0$ is calculated with the aid of (1)). The energy gap, determined with the aid of the equation $\omega = \Sigma(-i\omega)$, is equal to

$$\frac{\Delta}{T}\Big|_{T\to T_k} = \alpha\left(1-\frac{T}{T_k}\right)^{1/2}.$$

In the weak-coupling approximation, as is well known, $\alpha = 3.06$. In our case $\alpha$ is determined during the course of the solution of (3); the corrections turn out to be of the order of $(\pi T_k/\theta)^2$. A quantitative calculation was made for Pb. We used the experimental data describing the phonon spectrum of Pb [5]. The calculation was made in the Einstein model, and also for the case when the phonon spectrum of Pb is approximated in accordance with [6] (we assume that $\omega = uq|_{0<q<0.35q_D}$; $\omega = \omega_0 = 0.7\theta|_{0.35q_D<q<q_D}$; $q_D$ - Debye momentum) (see also [7]). It should be noted, however, that the result depends little on the details of the phonon spectrum. In particular, close results are obtained when considering an acoustic dispersion law in the entire momentum region.

Calculation leads to the following result for Pb:

$$\frac{\Delta}{T}\Big|_{T\to T_k} \simeq 4\left(1-\frac{T}{T_k}\right)^{1/2}. \tag{4}$$

It is easy to show that allowance for the Coulomb interaction, with a numerical value, say, $g_{Coul}\nu = 0.11$ [2], increases $\alpha$ somewhat.

In calculating the heat capacity we start from the usual equation for the entropy, which reduces also in the Froehlich model to the form

$$S_{T\to T_k} = \frac{\pi^2}{3}\nu T\left[1-\frac{3}{2\pi^2}\left(\frac{\Delta}{T}\right)^2\right]$$

($\nu$ - density of state on the Fermi surface), giving for the jump in the heat capacity

$$\beta = C_s(T_k)/C_n(T_k) = 1 + \frac{3}{2\pi^2}\alpha^2.$$

Substituting the value of $\alpha$ determined from (4), we get $\beta_{Pb} \simeq 3.4$, which fits satisfactorily the experimental data (according to [1], $\beta_{Pb} = 3.65$, and according to [8] (see also [9]) $\beta_{Pb} = 3.4$.

It can be shown by starting from the exact definition of the thermal-conductivity coefficient $\kappa$ [10] that the function $\kappa(T)$ is determined by the same formulas as are obtained in the ordinary theory [11], except that $\Delta(T)$ is defined as in (4). Thus, for example, the electronic thermal conductivity $\kappa_e$, which is determined by impurity scattering, is given by the following formula [12]:

$$\kappa_{es}/\kappa_{en} = F(T)/F(T_k); \quad F(T) = \theta^{-1}\int_{-\infty}^{\Delta}\epsilon^2 \frac{\partial f_0}{\partial \epsilon}\, d\epsilon,$$

which, when (4) is taken into account, leads to a steeper decrease of $\kappa_e$ with decreasing temperature than in the usual case; this is indeed observed experimentally. Similar results are obtained also in the analysis of the thermal conductivity of pure anomalous superconductors. With this, the experimental data of [12] are described satisfactorily by the formulas obtained in [13] with allowance for (4).

It should be noted that anomalous superconductors can be produced synthetically. As is well known, when heavy impurities are introduced into a crystal, a quasilocal peak appears in the crystal phonon spectrum, with a frequency $\omega_m/\sqrt{(M/m - 1}$ ($\omega_m$ and m are the maximum frequency and the mass of the matrix atom, and M is the mass of the impurity atom) [14]. Because of this, the effective Debye temperature, which determines the critical temperature in accord with the formula $T_c = 1.14\theta e^{-1/g}$, becomes smaller when the heavy impurity is added. At the same time, in the case of a sufficiently large concentration of a nonmagnetic impurity (10 - 20%), the additional attraction between the conduction electrons, which results from their interaction with the electrons at the impurity levels, may cause the constant g to increase (see [15]). Then the ratio $T_c/\theta$ increases compared with a pure superconductor, corresponding to the appearance of strong coupling.


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## SUPERFLUIDITY OF THE COSMOLOGICAL NEUTRINO "SEA"

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In cosmological models, where states with very high density exist, the neutrino plays a very important role, if only from the point of view of formation of several chemical elements. The influence of the neutrino is particularly important in models in which a degenerate neutrino "sea" exists [1], and also apparently in the presence of considerable anisotropy during the earlier evolution stages [2].

In the isotropic model (the Friedmann model) at the earlier stages of the evolution, the total density is

$$\rho = \frac{\epsilon}{c^2} = \frac{3}{32\pi G t^2} = \frac{4{,}5\cdot 10^5}{t^2}\ \text{g/cm}^3$$

(see [3]), where t is the time reckoned from some initial instant. At the same time, the density of completely degenerate neutrinos (or antineutrinos) is

$$\rho_\nu = \frac{\epsilon_F^4}{8\pi^2 c^5 \hbar^3} \simeq 3\cdot 10^3 [\epsilon_F(\text{MeV})]^4 \text{g/cm}^3 ,$$

where $\epsilon_F$ is the energy at the Fermi boundary. It is obvious that $\rho_\nu \lesssim \rho$, and thus,

$$\rho_{\nu_{max}} = \frac{\epsilon_{F_{max}}^4}{8\pi^2 c^5 \hbar^3} \sim \frac{3}{32\pi G t^2}, \quad \epsilon_{F_{max}} \sim \frac{3}{\sqrt{t(\text{cek})}}\ \text{MeV}. \quad (1) \qquad (1)$$

We have assumed above, as usual, that the neutrinos form an ideal Fermi gas. Yet the neutrinos interact with one another, which under the conditions of a degenerate Fermi gas