in the region where the continual theory is valid, the order of magnitude of R is $a(B/|I|S^2z)^{1/5}$.

It should be noted that when S and R are small it is necessary to take into account the fluctuations of the conduction-electron spin outside the ferromagnetic region. The appropriate calculations will be reported in a separate article.

The foregoing analysis can be extended to include the case A < 0, by replacing A with |A|, if |A/B| is sufficiently small. In the opposite case, the s-electron spin cannot be regarded as fixed, and the wave function takes into account the oscillations of the d-electron spins in the ferromagnetic well, owing to their exchange with the s-electron. We then get

$$E_n \simeq E_A - \frac{1}{2}|A|(S+1) - |B|\left(\frac{2S}{1+2S}\right)\left[z - \frac{\pi^2 a^2}{R^2}\right]. \qquad (6)$$

As seen from (6), the second variant of the trial function, which ensures, unlike the first, a full gain of the s-d exchange energy, results at the same time in some loss of translation-energy motion.

A characteristic feature of the magnetic polaron is its anomalously large magnetic moment, which can exceed 100 Bohr magnetons. This moment should affect particularly storngly the longitudinal magnetic susceptibility of the antiferromagnet. An anomalous moment should also be possessed by the electron captured by the impurity center, but its value is smaller than that of the free magnetic polaron. Thermal ionization of the impurity centers should therefore lead to an increase of χ until the magnetic polarons begin to dissociate. At high densities of the alloying impurity, the increase of temperature can lead in principle to a transformation of the crystal into a ferromagnetic one with a simultaneous sharp increase of the electric conductivity.

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SELF-FOCUSING OF LIGHT IN SOLIDS VIA THE ELECTROSTRICTION MECHANISM

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The self-focusing of light is dealt with in [1-3]. The structure of the light beam in the case of self-focusing by electrostriction was investigated in detail in [3]. It will be shown here, however, that the results of [3] are not applicable even in an isotropic solid, and pertain only to a liquid. The point is that a deformation that is uneven over the volume of a solid, and the corresponding change in the refractive index, cannot be connected with only the local value of the electrostriction force, as is done in [3]. An investigation shows that if the self-focusing mechanism via electrostriction does exist in solids, then the beam in the channel will have a rather complicated polarization.

We consider for simplicity stationary self-focusing of unpolarized light in an isotropic medium. It is of importance to us that in such a problem we can confine ourselves to the case of axial symmetry for elastic deformation, and regard only the radial component of the displacement u_r as different from zero. Since the change in the dielectric constant $\delta\varepsilon_{ij}$ is connected under our assumptions only with the relative deformation u_{ij}, we can write

$$\delta\epsilon_{ij} = \alpha u_{ij} + \beta u_{kk}\,\delta_{ij}\,, \qquad (1)$$

where α and β are constants. The elasticity-theory equations for u_r are [4]:

$$\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(r u_r)\right] = -Af_r\,; \qquad A = \frac{(1+\delta)(1-2\delta)}{E(1-\delta)}\,. \qquad (2)$$

Here E is Young's modulus, δ the Poisson coefficient, and f_r the radial component of the forces produced in the medium by the light flux. The striction force can be readily determined in the following manner: In considering the elastic properties, we introduce a correction to the free energy, due to the presence of the light flux and including both the electric field E_i of the light wave and the deformation u_{ij}:

$$\delta F = a u_{ij}\,\overline{E_i E_j} + b u_{ii}\,\overline{E_j^2}\,, \qquad (3)$$

where a and b are constants and the superior bar denotes throughout averaging with respect to time. The striction force connected with the light wave

$$\left(f_i = \frac{\partial^2(\delta F)}{\partial u_{ik}\,\partial x_k}\right),$$

can then be easily expressed in our case in terms of the mean squares of the radial and angular components of the electric field $\overline{E_r^2}$ and $\overline{E_\phi^2}$:

$$f_r = \frac{\partial}{\partial r}\left[(a+b)\overline{E_r^2} + b\overline{E_\phi^2}\right] + \frac{a}{r}\left(\overline{E_r^2} - \overline{E_\phi^2}\right)\,. \qquad (4)$$

(We do not put $\overline{E_r^2} = \overline{E_\phi^2}$, since the properties of beams polarized normal and parallel to the radius turn out to be different.)

If a light beam of finite power forms a channel, then $\overline{E_r^2}$ and $\overline{E_\phi^2}$ should fall off at large distances from the symmetry axis more rapidly than r^{-2}. Then the solution of (2), with account taken of (4) is:

$$u_r = -\frac{A}{r}\int_0^r\left[(a+b)\overline{E_r^2} + b\overline{E_\phi^2}\right]r'\,dr' + \frac{Aa}{r}\int_0^r r'\,dr'\int_r^\infty \frac{\overline{E_r^2} - \overline{E_\phi^2}}{r''}\,dr''. \qquad (5)$$

From this we readily obtain the nonzero components of the relative deformation tensor:

$$u_{rr} = \frac{\partial u_r}{\partial r} \text{ and } u_{\phi\phi} = \frac{u_r}{r}\,.$$

Substituting the corresponding values in (1), we obtain expressions for $\delta\varepsilon_{rr}$ and $\delta\varepsilon_{\phi\phi}$ ($\delta\varepsilon_{r\phi} = \delta\varepsilon_{rz} = \delta\varepsilon_{\phi z} = 0$), consisting of terms that drop off not faster than r^{-2}, and for the terms falling off in accord with the same law as $\overline{E_r^2}$ and $\overline{E_\phi^2}$. These terms are apparently comparable at small values of r. For large r, however, only terms of the first type are significant, and we then obtain

$$\delta\epsilon_{rr} \simeq -\delta\epsilon_{\phi\phi} \simeq \frac{aA}{r^2}\int_0^r [(a+b)\,\overline{E_r^2} + b\,\overline{E_\phi^2}]\, r'\,dr' - \tag{6}$$

$$- \frac{aAa}{r^2}\int_0^r r'\,dr' \int_{r'}^{\infty} \frac{\overline{E_r^2} - \overline{E_\phi^2}}{r''}\,dr'' .$$

It follows directly from (6) that either the rays polarized along the radius or the rays polarized normally to the radius will not be retained in the channel. This means that only rays of one of these types can make up the channel.

We therefore consider for concreteness the case when $\delta\varepsilon_{\phi\phi} > 0$, i.e., the channel is made up by rays polarized normally to the radius. Since $\delta\varepsilon_{\phi\phi} \sim r^{-2}$ when r is large (see (6)), the question arises whether a solution with $\overline{E_\phi^2}$ falling off more rapidly than r^{-2} can exist for an electromagnetic wave with this $\delta\varepsilon_{\phi\phi}$. Putting $E(r, z) = E_0(r)\cos(k_z z - \omega t)$ (the existence of such a wave within the framework of the theory of [3] was considered in [5]), we obtain in analogy with [3]:

$$\frac{d^2E_0}{dr^2} + \frac{1}{r}\frac{dE_0}{dr} - \frac{E_0}{r^2} - (k_z^2 - \frac{\omega^2}{c^2}\epsilon_0)\,E_0 + \frac{\omega^2}{c^2}\,\delta\epsilon_{\phi\phi}\,E_0 = 0 . \tag{7}$$

Here ε_0 is the dielectric constant of the medium for a weak electromagnetic wave. When

$$k_z^2 - \frac{\omega^2}{c^2}\epsilon_0 > 0$$

and when r is so large that $\delta\varepsilon_{\phi\phi} \sim r^{-2}$, we obtain an equation for the modified Bessel function with real or pure imaginary index. In this case a solution exists in the form of a Macdonald function, which falls of as $r \to \infty$ like

$$r^{-1/2}\,\exp[-(k_z^2 - \frac{\omega^2}{c^2}\epsilon_0)^{1/2}\,r] .$$

Thus, in spite of the slow decrease of $\delta\varepsilon_{\phi\phi}$ with r, the intensity of the light beam can fall off exponentially. (An analysis of the self-focusing of a beam polarized radially will lead to an identical result.) We note that the latter consideration is not in any way a rigorous proof of the existence of self-focusing of a light beam via the electrostriction mechanism. It is necessary for this purpose to solve simultaneously the systems of electrodynamics and elasticity-theory equations.

It is important to note that even at the very start of the tunneling by the beam, very complicated polarization is observed inside the beam. Indeed, in the preceding analysis the only cause of light polarization was the anisotropy of the optical properties, which varied over the cross section. But since

$$u_{rr} = \frac{\partial u_r}{\partial r} \text{ and } u_{\phi\phi} = \frac{u_r}{r}$$

are likewise not equal to each other when u_r is not stationary (i.e., different degrees of contraction in the normal and radial directions), the optical properties of the radially and normally polarized beams differ (see (1)). This means that a complicated and variable polarization of the light beam will be observed from the very beginning of the formation of the channel.

Thus, the results of [3] cannot describe even stationary self-focusing of light in solids via the electrostriction mechanism. An important factor is that if stationary self-focusing does take place in this case, then not all the light rays will be retained in the channel, but only those having a definite polarization.

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COHERENT PHOTON DECAY IN A NONLINEAR MEDIUM

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The purpose of the present communciation is to estimate the possibility of observing a new type of light scattering, whereby the incident wave ("pump")

$$\frac{1}{2}\mathbf{E}_3 \exp[i(\omega_3 t - \mathbf{k}_3 r)] + \text{c.c.}$$

is transformed, as a result of the nonlinear polarizability of the medium, into a pair of waves

$$\frac{1}{2}\mathbf{E}_{1,2} \exp[i(\omega_{1,2} t - \mathbf{k}_{1,2} r)] + \text{c.c.},$$

satisfying the conditions $\omega_3 = \omega_1 + \omega_2$ and $\vec{k}_1 + \vec{k}_2 - \vec{k}_3 = \Delta = 0$. In the optical band, these conditions lead, as a result of the dispersion of the refractive index $n(\omega)$, to a practically single-valued (or doubly-valued) connection between the direction and the frequency of the scatterd radiation. This effect is similar tc spontaneous Mandel'shtam-Brillouin scattering, the second electromagnetic wave playing here the role of the acoustic wave. We confine ourselves to the case when $t\omega_{1,2} \gg kT$ and the medium is transparent at frequencies $\omega_{1,2}$, so that the scattering is connected only with quantum fluctuations [1]. These fluctuations determine the noise level of parametric light amplifiers and probably the efficiency of the existing pulsed parametric light generators, in which no stationary oscillation amplitude manages to establish itself.

In the case of low pump power (when the gain is $G \leq 1$ and the principal role is played by spontaneous decay) the scattered radiation can be called "parametric luminescence" with corresponding "parametric superluminescence" when $G > 1$. We present below formulas for the intensity S and the band $\Delta\omega$ of the frequencies emitted in a given direction in these two cases.

The spectrum of parametric luminescence ω is limited, besides the condition $\Delta = 0$, only by the condition that the crystal be transparent at the frequencies ω and $\omega_3 - \omega$.* For example,