BEHAVIOR OF THE DIFFRACTION PEAK FOR PARTICLES WITH ARBITRARY SPINS

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Logunov and Nguyen Van Hieu [1] have shown, by determining the width of the diffraction peak of processes in which scalar particles take part, namely a + b → a + b (I) and a + b → c + d (II)

$$\Delta^{I,II} = \sigma^{I,II}/(d\sigma^{I,II}/dt) \tag{1}$$

that $\Delta^{I,II}$ cannot decrease with increasing S (the square of the energy) more rapidly than const/$\ln^2 S$. We shall show in this paper that this behavior holds also in the case of particles with spins, and obtain explicit expressions for the constants preceding the indicated functions of S.

We consider first the elastic scattering processes (I). We denote by λ_i and λ_i' (i = a, b) the helicities of the initial and final particles. We expand the invariant helicity amplitudes in terms of partial waves as follows (see [2])

$$F_{\lambda_a\lambda_b\lambda_a'\lambda_b'}(S,t) = 8\pi\frac{\sqrt{S}}{k}\ \Sigma(2J+1)f^J_{\lambda_a\lambda_b\lambda_a'\lambda_b'}(S)d^J_{\lambda\mu}(\cos\theta), \tag{2}$$

where

$$\lambda = \lambda_a - \lambda_b; \quad \mu = \lambda_a' - \lambda_b',$$

$$d^J_{\lambda\mu}(\theta) = \frac{1}{2\lambda}\left[\frac{(J+\lambda)!(J-\lambda)!}{(J+\mu)!(J-\mu)!}\right]^{1/2} \times$$

$$\times (1+\cos\theta)^{\frac{\lambda+\mu}{2}}(1-\cos\theta)^{\frac{\lambda-\mu}{2}} P^{\lambda-\mu,\lambda+\mu}_{J-\lambda}(\theta) \tag{3}$$

for $\lambda \geqslant |\mu|$.

$$d^J_{-\lambda,-\mu}(\theta) = d^J_{\mu\lambda}(\theta) = (-1)^{\lambda-\mu}d^J_{\lambda\mu}(\theta).$$

As is well known, the presence of the singular factors $(1+\cos\theta)^{|\lambda+\mu|/2}(1-\cos\theta)^{|\lambda-\mu|/2}$ in the d-functions causes $F_{\lambda_a\lambda_b\lambda_a'\lambda_b'}$ to be non-analytic in z = cosθ in the region containing the segment [-1, 1].

We shall consider therefore amplitudes that are free of singular factors:

$$\tilde{F}_{\lambda_a\lambda_b\lambda_a'\lambda_b'}(S,z) = (1+z)^{-|\lambda+\mu|/2}(1-z)^{-|\lambda-\mu|/2}F_{\lambda_a\lambda_b\lambda_a'\lambda_b'}(S,z). \tag{4}$$

These amplitude are analytic in the Martin ellipse [3] with foci at z = ±1 and major axis $z_0 = 1 + 2\gamma/S$ ($\gamma > 0$). They can be represented in the form of series in the polynomials

$$e^{J}_{\lambda\mu}(z) = (1+z)^{-|\lambda+\mu|/2}(1-z)^{-|\lambda-\mu|/2}d^J_{\lambda\mu}(\theta) \tag{5}$$

in the following manner:

$$\tilde{F}_{\lambda_a\lambda_b\lambda_a'\lambda_b'}(S,z) = 8\pi\frac{\sqrt{S}}{k}\sum_J(2J+1)f^J_{\lambda_a\lambda_b\lambda_a'\lambda_b'}(S)e^J_{\lambda\mu}(\theta). \tag{6}$$

Applying the Cauchy formula to these functions $\tilde{F}_{\lambda_a\lambda_b\lambda_a'\lambda_b'}$ and repeating the entire reasoning of Greenberg and Low [4], we can show that $f^J_{\lambda_a\lambda_b\lambda_a'\lambda_b'}$ decrease exponentially when $J \to \infty$

$$|f^J_{\lambda_a\lambda_b\lambda_a'\lambda_b'}|^2 < R(S)[1 + 2\sqrt{\gamma/S}]^{-J}. \tag{7}$$

Marhoux and Martin have shown [5] that any regularized helicity amplitude satisfies dispersion relations with a finite number of subtractions in the circle $|t| < \gamma$. Using this result and applying the arguments of our earlier paper [6] we get

$$|f^J_{\lambda_a\lambda_b\lambda_a'\lambda_b'}|^2 < \operatorname{Im} f^J_{\lambda_a\lambda_b\lambda_a\lambda_b} \leqslant \text{const} \cdot S^{9/4}[1 + 2\sqrt{\gamma/S}]^{-J}. \tag{8}$$

We denote by J_0 the value of J at which the right side of (8) equals unity, $J_0 = 9/8(S^{1/2}/\gamma^{1/2}) \times \ln S$. Using the Schwartz inequality and the estimates

$$P_n^{(a,\beta)}(1) = \binom{n+a}{n}, \left| P_n^{(a,\beta)}(\theta) \right| \approx \frac{2}{\sqrt{\pi n}} \frac{1}{(\cos\theta/2)^{\beta+1/2}} \frac{1}{(\sin\theta/2)^{a+1/2}}$$

$$\theta \neq 0, \pi$$

(see formulas (4.11) and (8.21.10) of [7]), we can show that when $S \to \infty$ the diffraction peak satisfies the inequality

$$\left. \frac{1}{\sigma^I} \frac{d\sigma^I}{dt} \right|_{t=0} \leqslant 1 + \rho/2)^2 \frac{1}{\gamma} \ln^2 S.$$

$$\left. \frac{1}{\sigma^I} \frac{d\sigma^I}{d\cos\theta} \right|_{\theta \neq 0,\pi} \leqslant 1 + \frac{\rho}{2} \frac{S^{1/2} \ln S}{\pi \sin\theta \sqrt{\gamma}},$$

where ρ is a constant such that $\sigma^I \geq \text{const} \cdot S^{-\rho}$.

Let us consider the process (II). We expand the amplitudes in partial waves

$$F_{\lambda_a\lambda_b\lambda_c\lambda_d} = 8\pi \sqrt{\frac{S}{kk'}} \sum_J (2J+1) g^J_{\lambda_a\lambda_b\lambda_c\lambda_d} d^J_{\lambda\mu}(\theta)$$

It follows from the unitarity condition that

$$\operatorname{Im} f^J_{\lambda_a\lambda_b\lambda_a\lambda_b} = \sum_{\lambda_a'\lambda_b'} |f^J_{\lambda_a\lambda_b\lambda_a'\lambda_b'}|^2 + \sum_{\lambda_c\lambda_d} |g^J_{\lambda_a\lambda_b\lambda_c\lambda_d}|^2 + \dots .$$

We therefore have

$$|g^J_{\lambda_a\lambda_b\lambda_c\lambda_d}| \leqslant \sqrt{\operatorname{Im} f^J_{\lambda_a\lambda_b\lambda_a\lambda_b}} \leqslant \text{const} \cdot S^{9/8}[1 + \sqrt{\gamma/S}]^{-J}.$$

Repeating the same calculations as for the elastic processes, we get

$$\left. \frac{1}{\sigma^{II}} \frac{d\sigma^{II}}{dt} \right|_{t=0} \leqslant \frac{1}{\gamma} \left(1 + \frac{\rho'}{2}\right)^2 \ln^2 S,$$

$$\frac{1}{\sigma^{II}}\frac{d\sigma^{II}}{d\cos\theta}\bigg|_{\theta\neq 0,\pi} \leqslant \left(1+\frac{\rho'}{2}\right)\frac{S^{1/2}\ln S}{\pi\sin\theta\sqrt{\gamma}},$$

where ρ' is a constant such that $\sigma^{II} \geq \text{const}\cdot S^{-\rho'}$.

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TRANSVERSALITY OF FIELDS OF VECTOR AND AXIAL-VECTOR MESONS AND HELICITY SYMMETRIES

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We establish in this paper, on the basis of the Lagrangian formalism, the connection between the condition of the transversality of the fields of V and A mesons with helicity symmetries. We show that in theories in which the V and A fields remain transverse also in the interactino, a helicity group arises and moreover the field algebra is satisfied.

We consider the most general relativistically- and P-invariant Lagrangian with dimensionless coupling constants, describing the interaction of a generally arbitrary and different number of fields

$$P^a(0^-),\ \sigma^e(0^+),\ V^i_\mu(1^-),\ A^m_\mu(1^+),\ B^r(1/2^+) \tag{1}$$

and we construct the corresponding equations of motion. We assume, in addition, that

$$\partial_\mu V^i_\mu = 0,\ \ i=1,\dots,n_V;\ \partial_\mu A^m_\mu = 0,\ m=1,\dots,n_A. \tag{2}$$

The subsequent analysis is based on a general remark [1], according to which the equations of motion should not produce excessive limitations[1)] on the number of degrees of freedom of the fields (1). Therefore each term of the independent Lorentz structure in the additional conditions

$$(m^2_V)_{ij}\,\partial_\mu V^j_\mu = R^i(P,\sigma,V,A,B) = 0,$$
$$(m^2_A)_{mn}\partial_\mu A^n_\mu = Q^m(P,\sigma,V,A,B) = 0$$

(obtained by taking the 4-divergences of the equations of motions of the V and A fields and replacing in the resultant expression the higher-order derivatives of the fields (1) in accordance with the equations of motion, and then using conditions (2)) should vanish as a result of the coefficient. This yields a number of relations for the mass matrices and the

[1)] Except for the necessary limitations, such as the transversality conditions (2) and the equations of motion of the fields V_μ and A_μ themselves (at $\mu = 4$).