In conclusion, we note that a sufficiently ideal electronic system ($n^{-1/3}e^2 \gtrsim T$), even if it does not form a crystal, can apparently have a shear modulus at sufficiently large frequencies ω, such that the short-range-order realignment does not have time to occur within one period of the oscillations. Then passage of transverse electromagnetic waves with $\omega < \omega_0$ should be observed at these frequencies (as in an "electronic crystal").

[1] D. Pines, Elementary Excitations in Solids, Benjamin, 1963, Ch. III (Russ. Transl. Mir, 1965).
[2] R. A. Coldwell-Horsfall and A. A. Maradudin, J. Math. Phys. 1, 395 (1960).
[3] W. J. Carr, Phys. Rev. 122, 1437 (1961).

CRITICAL CURRENT OF A SUPERCONDUCTING FILM IN A MIXED STATE

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All papers dealing with the critical current of rigid superconductors contain the statement that the mixed state ($H_{c1} < H_0 < H_{c2}$) is absolutely unstable with respect to a transport current directed perpendicular to the external magnetic field H_0. In other words, this means that the transport current interacting with the superconducting vortices exerts on them Lorentz force and causes them to move in a direction perpendicular to the field and to the current. This gives rise to energy dissipation and destruction of the superconducting state. If the material is inhomogeneous, then the vortices become pinned to the inhomogeneities and nondissipative superconducting flow of the transport current is possible.

We note first of all that the statement that the mixed state is absolutely unstable is, strictly speaking, incorrect. Indeed, it would be valid for an infinite sample, but even the surface of a real sample can serve as the homogeneity on which the vortex motion can become pinned.

We calculate in this paper the critical current for a film placed parallel to an external magnetic field. The film thickness d is assumed small compared with the penetration depth δ_0: $d \ll \delta_0$, but $d \gg \xi(T)$, where $\xi(T) = \delta_0/\kappa$ and $\kappa \gg 1$ (κ is the constant of the Ginzburg-Landau theory [1]). We consider a case when the external magnetic field is $H_0 > H_{c1}(d)$, but $H_0 - H_{c1}(d) \ll H_{c1}(d)$. Here $H_{c1}(d)$ is the first critical field of the film, which was calculated by Abrikosov in [2]. Let the film be parallel to the (yz) plane and bounded by the planes $x = \pm d/2$. The external magnetic field is directed along the oz axis, and the transport current flows in the oy direction.

We consider first the case when there is no transport current. Let us find the free energy of such a configuration of vortices: the axes of all the vortices are parallel to yz, the points of intersection of the axes of all the vortices with the (xy) plane lie on a single line parallel to the oy axis and located a distance x_0 away from it, and the distance a between the axes of the vortices is large compared with δ_0. The solution of the equation for the field

$$\Delta H - H = -\frac{2\pi}{\kappa}\sum_{m=-\infty}^{\infty}\delta(x - x_0)\,\delta(y - ma)$$

under the boundary condition $H = H_0$ at $x = \pm d/2$ is obtained in the manner used in [2]. We use the relative units introduced in [1] throughout. Substituting the solution in the expression for the free-energy density F_H, we obtain the free-energy density as a function of the parameter x_0. As expected, this function has a minimum at $x_0 = 0$, i.e., it is most convenient for the chain of vortices to arrange itself in the center of the film (as $a \to \infty$). The obtained expression makes it possible to find the restoring force acting on the chain of vortices: $f = - dF_H/dx_0$. Let now a transport current flow through the film, with a distribution determined by the solution of the London equation with the boundary condition $H = \pm H_1$ at $x = \pm d/2$. This transport current exerts on the vortex a Lorentz force f_L, which shifts the chain of vortices a distance x_0. The condition for the equilibrium of the vortices is obvious: $f_L = dF_H/dx_0$. This equation establishes the following dependence of H_1 on x_0:

$$H_l = \frac{1}{\kappa d} \ln \frac{d/2 + x_0}{d/2 - x_0} \frac{\mathrm{sh}\, d/2}{\mathrm{ch}\, x_0} - H_0 \mathrm{th}\, d/2\ \mathrm{th}\, x_0. \qquad (1)$$

In the derivation of this equation we neglect the interaction of the vortices, since we assume that $a \to \infty$. It is seen from (1) that H_1 is an increasing function of the $-x_0$ only up to a certain value x_{0c}, which determines the stability limit of the vortex lattice. Thus, x_{0c} is determined from the equation $dH_1/dx_0 = 0$, and the obtained value of x_{0c} is substituted in (1). This determines the critical current.

In the approximation in which $x_{0c} << d/2$, we obtain the following expression for the magnetic field produced on the surface of the film by the critical current

$$H_{l_c} = \frac{1}{12} \sqrt{\kappa}\, d^3 [H_0 - \frac{H_{c1}(d)}{\ln \gamma \kappa d/\pi + 0{,}081}]^{3/2}, \qquad \gamma = e^C = 1{,}78. \qquad (2)$$

It follows from this expression that the critical current increases with increasing magnetic field $H_0 (H_0 > H_{c1}(d))$. Physically this is perfectly understandable. With increasing external magnetic field, the potential well in which the chain of vortices is situated becomes deeper and a larger transport current is necessary to upset the stability of this system. On the other hand, it is clear that the superconductivity of the film is completely destroyed in any case when $H_0 = H_{c2}$. It follows therefore that the critical current reaches a maximum at some field H_p ($H_{c1}(d) < H_p < H_{c2}$). We thus arrive at the conclusion that a peak effect should be observed for the films under consideration (a peak in the dependence of the critical current on the external magnetic field).

[1] V. L. Ginzburg and L. D. Landau, Zh. Eksp. Teor. Fiz. 20, 1064 (1950).
[2] A. A. Abrikosov, ibid. 46, 1464 (1964) [Sov. Phys.-JETP 19, 988 (1964)].

ERRATUM

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On p. 219, the second formula of (2) reads "...$(\gamma_{3s_2} + \gamma_{2p_4})$..." should read $(\gamma_{3s_2} - \gamma_{2p_4})$.." On the same page, line 8 from the bottom reads " H. Holt [7],..." should read "H. Holt [8],..." and on the third line from the bottom reads "...transition [6]..." should read "transition [7]." The following additional reference should be added: [8], H. K. Holt, Phys. Rev. Lett. 20, 410 (1968)].