Jupiter.

Let us consider further the precession of the proper angular momentum of the earth (E) under the influence of its own orbital motion ("spin-orbit interaction").

According to the magnetic analogy we have for the torque acting on the earth

$$K_E = \dot{M}_E = [M_E \times H_E]. \tag{10}$$

Here H_E can be represented as the intensity of the magnetic field produced by a linear current whose direction coincides with the velocity of the orbital motion v_E.

From (10) it follows with the aid of (5) that M_E precesses with an angular velocity

$$|\Omega_{pr}| = \frac{2G}{C^2}\frac{m_E V_E}{\pi R_E r_E} \approx 8{,}8\cdot 10^{-17}\ \mathrm{sec}^{-1}. \tag{11}$$

Here R_E is radius of the earth's orbit, and m_E and r_E are the mass and radius of the earth.

According to (11), the axis of the earth tilts ~0.06" per century. Such quantities can be measured in principle, but the situation is aggravated by the relatively large precession of the earth in accordance with Newton's laws (50.24" per year) under the influence of the attraction of the moon, sun, and the planets.

In conclusion, the author thanks Ya. B. Zel'dovich for suggesting the use of the magnetic analogy for an estimate of the effect under consideration, and A. G. Doroshkevich for useful discussions of the work.

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ABSORPTION AND SCATTERING OF LIGHT BY EXCESS ELECTRONS IN LIQUID HELIUM

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Submitted 11 July 1967
ZhETF Pis'ma 6, No. 6, 715-718 (15 September 1967)

An investigation of the mechanism of charge motion in liquid helium has shown that a cavity of approximate radius 20 Å is produced in the helium around an excess electron. To the electron, the cavity represents a potential well of approximate depth 1.3 eV. Such a well can accommodate several (~10) electron levels. If a beam of light passes through liquid helium containing excess electrons, effects connected with transitions of the electrons between the levels, namely absorption of resonant frequencies and scattering, can be observed. Some ideas concerning the optical properties of excess electrons in helium were already advanced in [1]. In this letter we present estimates of the magnitudes of these effects.

The "electron inside a bubble" system is analogous in many respects to a molecule. The nuclear motion corresponds here to deformation of the bubble, the periods of the nuclei are much larger than the periods of the electronic motions, so that we can assume that the system of electronic terms "follows" the variation of the shape and dimension of the bubble. To each electronic state there corresponds its own equilibrium dimension and shape of the bubble, and these are obtained by minimizing the energy of the system. To estimate the order of magnitude of the frequencies of the deformation oscillations of the bubble, we considered the simplest case - spherically symmetrical oscillations. When the electron is in the ground state, the frequency of these oscillations is $\Omega \approx 4 \times 10^{10}$ sec^{-1}. In calculating the frequencies of the absorption lines and the values of the absorbed energy, the deformational oscillations of the bubble can be disregarded, i.e., it can be assumed that the transitions occur in a spherical well with a radius corresponding to the ground state of the electron. Allowance for the bubble deformations is essential only for the determination of the line width.

Noticeable effects arise only as a result of transitions between the lower levels, for which the well can be regarded as square and infinitely deep. The energy of the n-th electronic state with angular momentum l, reckoned from the bottom of the well (see [2]) is $E_{nl} = \hbar^2\beta_{nl}^2/2ma^2$.

Here m is the electron mass, a the radius of the well (for the chosen model, $a \simeq 19$ Å), and β_{nl} is the n-th root of the Bessel function of order $l + 1/2$. The sequence of levels in an infinitely deep well is 1s, 1p, 1d, 2s, 1f, 2p, 1g, 2d, 1h... The absorption of light may result from a transition of the electron from the ground state 1s to a state with angular momentum $l = 1$, i.e., 1p or 2p. Other transitions are forbidden by the selection rules. We can actually speak only of the 1s → 1p transition, since the matrix element of the transition in the state 2p is much smaller. The frequency of the light absorbed in the 1s → 1p transition is $\omega_{1p}^{1s} = (E_{1p} - E_{1s})/\hbar \simeq 1.6 \times 10^{14}$ sec^{-1}, corresponding to a wavelength $\lambda = 12\ \mu$. This is the infrared band.

The total energy S absorbed per unit time and per unit electron in the transition from the state a into the state b is determined from the formula [3]

$$S = \frac{4\pi^2}{3}\frac{e^2}{\hbar e}\,|(r)_b^a|^2\,\omega_b^a\,I_0\,(\omega_b^a).$$

Here $\omega_b^a = (E_b - E_a)/\hbar$, $I_0(\omega)$ is the spectral density of the intensity of the incident light, $e^2/\hbar e = 1/137$, and $(r)_b^a$ is the matrix element of the radius vector of the electron between the initial and final states, which can be readily calculated for the chosen model: $|(r)_{1p}^{1s}| \approx 0.53$ a. In the cgs system $S \approx 0.16\ I_0(\omega_{1p}^{1s})$.

A preliminary estimate yields for the line width γ a value $\sim 10^{12}$. This is essentially the temperature broadening due to transitions in which the radius of the bubble changes. The coefficient of the absorption of light by extraneous electrons in helium, in a frequency interval γ with center at ω_{1p}^{1s}, is then defined as $\nu S/I_0(\omega_{1p}^{1s})\gamma$, where ν is the concentration of the excess electrons. Its order of magnitude is 1% at a concentration 10^{11} cm^{-3}; such concentrations can apparently be obtained with the existing radioactive sources. The greater part of the absorbed energy will be radiated in the form of resonantly-scattered light.

Raman scattering can also be produced in such a system. The most intense line will correspond to the transition 1s → 1d, and the frequency shift of the scattered light will be $\omega_{1d}^{1s} \simeq 3.8 \times 10^{14}$ sec^{-1} and will be in the direction of the lower frequencies. The effective cross section $\sigma(\theta)$ of Raman scattering of light of frequency ω by one electron can be calculated by means of the general formulas [4]. In our case we can confine ourselves to allowance for only one intermediate state 1p, the contributions of the remaining states being small, and then

$$\sigma(\theta) = \frac{(\omega+\omega_{1d}^{1s})}{2c^2}\left[\frac{\omega_{1p}^{1s} + \omega_{1p}^{1d}}{(\omega_{1p}^{1s}-\omega)(\omega_{1p}^{1d}+\omega)}\right]^2 \left(\frac{\ell^2}{\hbar c}\right)^2 a^4 \left[\left(\frac{r}{a}\right)_{1p}^{1s}\left(\frac{r}{a}\right)_{1d}^{1p}\right]^2 \frac{6+\sin^2\theta}{45}.$$

Here θ is the angle between the direction of incidence of the light and the observation direction, r denotes the absolute value of the radius vector of the electron, and the matrix elements are taken between the radial parts of the corresponding wave functions, for which the values obtained are $(r/a)_{1p}^{1s} \simeq 0.53$ and $(r/a)_{1d}^{1p} \simeq 0.59$. If the source of light is a ruby laser, then $\sigma(\theta) \simeq 2.5 \times 10^{-26}(6 + \sin^2\theta)/45$ cm^2.

We have assumed so far that the temperature of the helium is 1 - 2°K and there is no external pressure. When pressure is applied, the radius of the bubble decreases, and the transition frequencies increase. Corresponding to the 1s → 1p transition is a frequency 2.8×10^{14} sec^{-1} (λ = 7 μ) at 10 atm and 3.6×10^{14} sec^{-1} (λ = 5 μ) at 20 atm.

The author is grateful to L. P. Pitaevskii for suggesting the consideration of the questions touched upon in this letter and for useful discussions, and also to A. I. Shal'nikov and I. I. Abrikosova for a discussion of the experimental possibilities.

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E R R A T A

Article by I. A. Fomin, Vol. 6, No. 6

Page 197, line 3 from the bottom, reads: "This is essentially the temperature broadening due to transitions..." It should read: "This is essentially the temperature broadening and the broadening due to transitions..."

In article by I. A. Fomin, Vol. 6 No. 6, p. 198, line 2, read "... 12^{-2} 1/cm..." in lieu of "... 1%..."