of values of a_0 and a_2 for which $\chi^2 \leq 6$ (in the comparison of the corresponding theoretical formulas with the spectrum of Fig. 1c, the number of degrees of freedom is equal to 6).

The kinetic energy released in the $K \to 3\pi$ decay is not too large (~80 MeV), so that one might ask whether terms of order E^2, $E^{5/2}$, etc. could play a significant role. There are, however, considerations indicating that these terms are small. First, it follows from the theoretical formulas that the spectra $W^{++-}(\epsilon)$ and $W^{++-}(Z)$ should be described by formulas (1), which is in good agreement with the available experimental data. Second, the theoretical formulas [4] predict that when $\epsilon = 1$ there should be deviations of $W^{00+}(\epsilon)$ and $W^{+-0}(\epsilon)$ from linearity, towards larger values, for arbitrary nonvanishing a_0 and a_2. This is also observed experimentally (see Fig. 1). If the terms of order E^2, $E^{5/2}$, etc. are not small, there would be no special reason to expect such a behavior of the energy spectra $W^{++-}(\epsilon)$, $W^{++-}(Z)$, $W^{00-}(\epsilon)$, and $W^{+-0}(\epsilon)$.

The theoretical formulas [4] take into account terms of order unity, E, and $E^{3/2}$. The extent to which the region of the permissible values of a_0 and a_2 changes can be assessed by neglecting in these formulas the terms of order $E^{3/2}$ (retaining only the terms of order unity and E). It turns out that in this case the possible values of a_0 and a_2 (values leading to $\chi^2 \leq 6$) lie in the interval $0.75 \leq |a_0 - 1.1a_2| \leq 1.2$. We see thus that in this case the terms of higher order in E (of order $E^{3/2}$) have no strong influence on the result.

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CALCULATION OF THE WIDTH OF THE $f^0 \to 2\gamma$ DECAY FROM THE DISPERSION SUM RULES

G. M. Radutskii

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Great interest attaches to investigations of radiative decays of hadrons in connection with checks on the relations that follow from the assumed existence of higher symmetries. These symmetries can be very strongly violated, as indicated, for example, by the situation with the relation between $\Gamma(\eta \to 2\gamma)$ and $\Gamma(\pi^0 \to 2\gamma)$ [1]. It is therefore of interest to obtain information on the radiative decays from a reasonable dynamic model, without using the arguments of symmetry theory.

To obtain information on hadron properties, use has been made recently of dispersion sum rules derived on the basis of the dispersion relations under the assumption that the amplitudes decrease sufficiently rapidly at high energies [2]. In this paper we use the dispersion sum rule for the amplitude of the reaction $\gamma + \gamma \to \pi^0 + \pi^0$ to calculate the width of the $f^0 \to 2\gamma$ decay. This quantity has already been obtained in [3], but the value $\Gamma_{f\to 2\gamma}/\Gamma_f \approx 6\%$ derived in that paper seems too high to us.

The invariant amplitude of the reaction $\gamma + \gamma \to \pi^0 + \pi^0$ can be written in the form [4]

$$T = D_\alpha(t, s, u) I_\alpha + D_\beta(t, s, u) I_\beta ,$$

where

$$I_\alpha = (e_1 e_2) - \frac{(e_1 k_2)(e_2 k_1)}{(k_1 k_2)} .$$

$$I_\beta = (e_1 \Delta)(e_2 \Delta)(k_1 k_2) - (e_1 k_2)(e_2 \Delta)(k_1 \Delta) - (e_2 k_1)(e_1 \Delta)(k_2 \Delta) + \frac{(e_1 k_2)(e_2 k_1)(k_1 \Delta)(k_2 \Delta)}{k_1 k_2} .$$

$(k_1 e_1)$ and $(k_2 e_2)$ are the 4-momenta and 4-vectors of polarization of the photons, $\Delta = p_1 - p_2$ is the difference between the pion 4-momenta,

$$s = (k_1 + p_1)^2 = -q^2 - \omega_q^2 - 2q\omega_q \cos\phi ,$$

$$u = (k_1 + p_2)^2 = -q^2 - \omega_q^2 + 2q\omega_q \cos\phi ,$$

$$t = (k_1 + k_2)^2 = 4p^2 = 4\omega_q^2 ,$$

$\vec{p}$ is the photon momentum and $\vec{q}$ is the pion momentum in the c.m.s.

It can be shown [5] on the basis of the limitations imposed by the unitarity condition that when $t \to \infty$ and $\cos\varphi \neq \pm 1$ we have

$$|D_\beta(t, s, u)| \lesssim t^{-5/4} .$$

This makes it possible to write the dispersion sum rule for this amplitude, by using the dispersion relation in t for fixed s, such that

$$\int_{-\infty}^{+\infty} \mathrm{Im}\, D_\beta(t, s)\, dt = 0.$$

We shall saturate this sum rule with the presently known resonances, using the zero-width approximation. It is easy to see that the 0^+ meson makes no contribution to $\mathrm{Im}\, D_\beta(t, s)$. The contribution from the $\varphi \to \pi\gamma$ vertex is very small. (In the quark model this vertex is completely forbidden [6].) Therefore the contribution of the φ meson will henceforth be disregarded. We also disregard the contribution of 1^+ mesons with negative charge parity, since there is presently no information on the widths of their radiative decays, which are apparently small.

Thus, takin account in the unitarity condition of only f, ρ, and ω mesons, we obtain the following relation for the interaction constants:

$$\frac{g_{f\pi\pi}\, g_{f\gamma\gamma}}{m_f^2} + \frac{g^2_{\rho\pi\gamma}}{4m_\rho^2} + \frac{g^2_{\omega\pi\gamma}}{4m_\omega^2} = 0,$$

which does not depend on cos φ in the approximation under consideration.

If we calculate from this equation $\Gamma(f \to 2\gamma)$, assuming that $\Gamma_{\omega\to\pi\gamma}$ = 1.15 MeV, $\Gamma_{\rho\to\pi\gamma}$ = 0.6 MeV, and $\Gamma_{f\pi\pi}$ = 100 MeV [7], then we get

$$\Gamma(f \to 2\gamma) = 0.02 \text{ MeV},$$

corresponding to approximately 0.02% of the total f-meson width.

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PHOTOPRODUCTION OF PSEUDOSCALAR MESONS ON NUCLEONS AT HIGH ENERGIES, AND REGGE POLES

M. P. Rekalo
Physico-technical Institute, Ukrainian Academy of Sciences
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We discuss in this note the consequences of the Regge-pole hypothesis for reactions of photoproduction of pseudoscalar mesons on nucleons. According to present-day notions, the behavior of the amplitudes at high energies and low momentum transfers can be described by taking into account exchange of the nonet of vector and nonet of tensor Regge pole (exchange the Pomeranchuk vacuum pole is forbidden in the photoproduction of pseudoscalar mesons). It is assumed here that the resultant interaction constants obey the relations of exact SU(3) symmetry, and the corresponding constants of interaction of the tensor and vector mesons coincide [1] (the principle of degeneracy of vector and tensor Regge poles). An analysis of the total cross section of hadron interaction has shown that the degeneracy principle holds with good accuracy.

We then have for the π- and η-meson photoproduction amplitudes:

$$F_\lambda(\gamma p \to p\pi^0) = \kappa g_\lambda (4 + 6\beta) R_\rho \zeta_-, \quad F_\lambda(\gamma n \to n\pi^0) = \kappa g_\lambda (2+6\beta) R_\rho \zeta_-,$$

$$F_\lambda(\gamma p \to p\eta) = \kappa g_\lambda \frac{8 - 8f + 6\beta}{\sqrt{3}} R_\rho \zeta_-, \quad F_\lambda(\gamma n \to n\eta) = \kappa g_\lambda \frac{2 - 8f + 6\beta}{\sqrt{3}} R_\rho \zeta_-, \tag{1}$$

$$F_\lambda(\gamma p \to n\pi^+) = \kappa g_\lambda \sqrt{2} R_\rho (\zeta_- - 3\zeta_+), \quad F_\lambda(\gamma n \to p\pi^-) = \sqrt{2}\kappa g_\lambda R_\rho (\zeta_- + 3\zeta_+),$$

and the K-meson photoproduction amplitudes are given by