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TEST OF BJORKEN'S ASYMPTOTIC FORMULA IN PERTURBATION THEORY

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In a recent paper [1], Bjorken proposed a formula for the asymptotic behavior of the matrix element $M_{\mu\nu}(p', k; p, q)$:

$$M_{\mu\nu}(p', k; p, q) = -i\int dx\, e^{ikx} \langle b|T\{V_\mu^+(x),\ V_\nu^-(o)\}|a\rangle\ , \tag{1}$$

corresponding to the amplitude for the scattering of a W boson by an arbitrary target, due to the weak vector current V^+. According to Bjorken, when $k_0 \to \infty$ and k, p, and p' are fixed, the matrix element (1) can be expressed in terms of the simultaneous current commutator with the aid of the equality

$$\lim_{k_0\to\infty} k_0 M_{\mu\nu}(p', k; p, q) = \int d\mathbf{x}\, e^{-i\mathbf{k}\mathbf{x}} \langle b|[V_\mu^+(o, \mathbf{x}), V_\nu^-(o)]|a\rangle. \tag{2}$$

Relation (2) can be derived in the following manner: We sum in (1) over the intermediate states and integrate with respect to k_0. We then get

$$\int d\mathbf{x}\, e^{-i\mathbf{k}\mathbf{x}} \sum_n \left\{ \frac{\langle b|V_\mu^+(o,\mathbf{x})|n\rangle\langle n|V_\nu^-(o)|a\rangle}{k_0 + p_0' - p_{n0}} - \frac{\langle b|V_\nu^-(o)|n\rangle\langle n|V_\mu^+(o,\mathbf{x})|a\rangle}{k_0 - p_0 + p_{n0}} \right\}. \tag{3}$$

Letting k_0 go to infinity in (3) and neglecting p_{n0} compared with k_0, we arrive at (2).

We note that in the derivation it is possible to substitute for V_μ any other operator, so that relation (2) seemingly applies to any two operators. In fact, however, it is clear that (2) is valid only when arbitrarily large intermediate energies p_{n0} play no role in the sum of (3) and in the sum produced when k_0 is taken out. This means that the amplitude $M_{\mu\nu}$ and the matrix element of the simultaneous commutator should be finite.

Let us consider by way of an example a field-theory model in which there are only nucleons and neutral pseudoscalar mesons, and the Lagrangian of their interaction is equal to

$$L = ig\bar{\psi}\gamma_5\psi\phi. \tag{4}$$

The vector and axial currents are defined by

$$V^{\pm}_{\mu}(x) = \bar{\psi}(x)\gamma_{\mu}\tau^{+}\psi(x)\,,$$
$$A^{\pm}_{\mu}(x) = \bar{\psi}(x)\gamma_{\mu}\gamma_{5}\tau^{\pm}\psi(x). \tag{5}$$

Let us calculate in this model, in the g^2 approximation, the asymptotic expression as $k \to \infty$ for the matrix element $M_{\mu\nu}$ corresponding to the scattering of W^+ by a proton. After simple calculations we obtain, with logarithmic accuracy ($\ln k^2/m^2 \gg 1$)

$$M_{\mu\nu} = \bar{u}(p')\,\{\gamma_{\nu}\hat{k}^{-1}\gamma_{\mu} - \frac{g^2}{8\pi}\ln\frac{k^2}{m^2}(\gamma_{\nu}\hat{k}^{-1}\gamma_{\mu} - \gamma_{\mu}\hat{k}^{-1}\gamma_{\nu})\,\}u(p), \tag{6}$$

where u(p) and u(p') are initial and final state spinors. Comparing (6) with (2), we can easily verify that relation (2) is satisfied for the components $M_{0\nu}$ and $M_{\mu 0}$ and is violated for the components M_{ik} ($i, k = 1, 2, 3, i \neq k$). * When $i \neq k$ the simultaneous commutator is

$$[V_i^+(x)\,V_k^-(0)]_{\substack{x_0=0\\ i\neq k}} = 2i\epsilon_{ik\ell}A_{\ell}(0)\,\delta(\mathbf{x}), \tag{7}$$

where $A_{\ell}(x)$ are the spatial components of the axial isoscalar current. Substituting (7) in (2) and calculating the g^2-approximation corrections to the axial vertex, we obtain from (2) and expression for M_{ik} which differs from (6) in that $\ln k^2/m^2$ is replaced by $\ln \Lambda^2/m^2$, where Λ is the cutoff momentum. Apparently the violation of (2) is due to the fact that the matrix element of the simultaneous commutator is infinite in this case. By virtue of the vector-current conservation, the amplitude is finite and is determined by an integral that converges at momenta on the order of k. Relation (2) is therefore valid when $\Lambda \sim k$.

We now consider another example in the same model, namely scattering of a W^+ boson by a proton due to axial current. In the same g^2-approximation and at large k, the matrix element for this process takes the form

$$M^{A}_{\mu\nu} = \bar{u}(p')\,\{(1 - \frac{g^2}{2\pi}\ln\frac{\Lambda^2}{m^2})\gamma_{\nu}\hat{k}^{-1}\gamma_{\mu} + \frac{g^2}{8\pi}\ln\frac{k^2}{m^2}(3\gamma_{\nu}\hat{k}^{-1}\gamma_{\mu} + \tag{8}$$
$$+ \gamma_{\mu}\hat{k}^{-1}\gamma_{\nu})\}\,u(p).$$

It follows from (8) that as $k_0 \to \infty$

$$M^{A}_{0\nu} = \frac{1}{k_0}\bar{u}(p')\,\gamma_{\nu}u(p)\,(1 - \frac{g^2}{2\pi}\ln\frac{\Lambda^2}{k^2})\,, \tag{9}$$

whereas an equality of the type (2) yields by virtue of the commutation relations

$$M^{A}_{0\nu} = \frac{1}{k_0}\,\bar{u}(p')\,\gamma_{\nu}u(p)\,. \tag{10}$$

(The right side of (2) is expressed in this case in terms of the vertex function $\Gamma_{\nu}(p', p)$, for which there are no corrections in our approximation, since the vector current is conserved.) The resultant discrepancy can be attributed in our case to the fact that the matrix element $M^A_{\mu\nu}$ (8) is not a finite quantity. If we renormalize $M^A_{\mu\nu}$, then the infinite renormalization constants will enter in the right side of a relation of the type (2), and the system becomes similar to that of the preceding example.

As a result, we arrive at the conclusion that in our approximation the Bjorken relation holds true whenever its right and left sides are finite quantities. In the case of weak vector and axial currents it is assumed in the theory of universal V-A interaction that the bare constant of this interaction is finite.** Since the axial-current renormalization constant is finite in V-A theory, the example considered by us offers evidence in favor of the applicability of Bjorken's relation to vector and axial currents in V-A theory. However, the application of these relations to any other field operator (say, the pion field operator φ), the renormalization of which can be infinite, is in general not valid.

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* Relation (2) for $M_{0\nu}$ is of the form $\lim k_0 M_{0\nu} = 2\Gamma^{(3)}_\nu(p', p)$, where $\Gamma^{(3)}_\nu$ is the vertex function; this relation follows directly from the relation $k_\mu M_{\mu\nu} = 2\Gamma^{(3)}_\nu$, in which vector-current conservation is used.

** We are dealing at all times with normalization due to strong interactions only.

MECHANISM OF GENERATION EXCITATION IN A CONTINUOUSLY OPERATING ARGON ION LASER

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The main processes that ensure population of the upper levels of a continuous argon ion laser are: (1) excitation by electron impact from the Ar^+ ground state [1,2], (2) from the 3rd configuration states of Ar^+, (3) radiative cascade population [2], and (4) direct electronic excitation from the ground state of the neutral Ar atom (Bennet's process) [3]. The lack of data on the ion excitation cross sections has not made it possible to compare the intensities of these processes directly, and there are only indirect experimental data [1-3].

We have calculated the rates of excitation $\langle v\sigma\rangle$ of the ions by electron impact, from the ground and the excited states (the angle brackets denote averaging over a Maxwellian velocity distribution). The cross sections were obtained numerically in a Born-Coulomb approximation, in which the external electron is described by continuous-spectrum Coulomb wave functions with $z = 1$. In this approximation, the effective cross section for the excitation of the configuration as a whole is practically independent of the coupling scheme of the atomic electrons. We did not calculate the cross sections for transitions between individual levels, since such a calculation calls for the use of an intermediate coupling scheme and is a rather laborious problem. We used in the calculations semi-empirical radial wave functions [4]. The obtained values of $\langle v\sigma\rangle$ are listed in the table.