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ON THE POSSIBILITY OF VERIFYING THE EQUIVALENCE PRINCIPLE (FOR THE ELECTROMAGNETIC FIELD) IN SECOND ORDER IN THE NEWTONIAN GRAVITATIONAL CONSTANT

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Submitted 28 May 1968
ZhETF Pis. Red. 8, No. 2, 110 - 113 (20 July 1968)

1. We have shown in [1] that violation of the equivalence principle for atomic phenomena could lead to a specific observable effect in strong gravitational fields. It turns out that violation of the equivalence principle (in second order in the Newtonian gravitational constant G) for electromagnetic phenomena could also lead to an observable effect, namely birefringence of light by a gravitational field. In the simplest case of violation of the equivalence principle, which we shall consider in Sec. 4, the gravitational birefringence would be appreciable even in so weak a gravitational field as the sun's. The optical path difference that this field produces between the polarization components of a light wave (from a nebula or a star) would have an appreciable magnitude, about 6×10^{-3} cm, i.e., 100 - 150 wavelengths in the visible band. An observer on earth could then discern seasonal fluctuations in the corresponding polarization planes, with amplitude up to 45°. Thus, an analysis of the results of systematic polarization measurements of light from nebulas and stars would greatly help in distinghishing between general relativity theory (GRT) and other relativistic gravitation theories that yield values for the three classical tests that agree with experiment; the proposed method supplements the one suggested by us in [1], and is simpler for observations.

2. We consider the equivalence principle in this paper in the same sense as in [1]. It follows from this principle, in particular, that no gravitational field should produce birefringence of electromagnetic waves, since the presence or absence of birefringence is invariant to the choice of the reference frame. Thus, in any theory that includes the equivalence principle for the electromagnetic field, the equations of this field (in the presence of gravitation) should have some singularity capable of excluding the appearance of gravitational birefringence, regardless of any anisotropy of the gravitational field, for example, non-equivalence of the radial and tangential directions in a spherically-symmetrical field. In the case of the equations of electrodynamics in GRT, such a singularity consists of the fact

that these equations can be reduced to the ordinary Maxwell's equations for continuous media, generally speaking moving media [2], and if the mixed components of the metric tensor vanish in the employed coordinate system, then we have an analogy with the electrodynamics of a medium at rest, and the corresponding tensors of the dielectric constant and the magnetic permeability are equal. Yet it is easy to show that the necessary and sufficient condition for birefringence in a medium at rest is the proportionality of the tensors $\varepsilon_{pq}(x)$ and $\mu_{pq}(x)$:

$$\mu_{pn}(x) = Z(x)\varepsilon_{pq}(x), \tag{1}$$

where $Z(x)$ is an arbitrary (smooth) function that neither vanishes nor becomes infinite; the indices p and q run through the values 1, 2, and 3; x denotes the set of four coordinates (x^0, x^1, x^2, x^3).

It follows from the foregoing that if the equivalence principle were to be violated, then we could expect as a consequence the occurrence, in some order n in G, of gravitational birefringence of electromagnetic waves. If furthermore the equivalence principle were to be violated in such a way that gravitational birefringence could occur also in a spherically-symmetrical gravitational field, then obviously in this case the difference Δv between the group (or phase) velocities of two types of waves corresponding (at some point M) to some spatial direction would depend only on the radial coordinate r of the point M, the angle α between the radius vector of the point M and the (say, group) velocity vector of the wave, and also on the mass m of the gravitating center and on the universal constants G and c. It follows from dimensionality considerations that this dependence would be of the form $\Delta v/c = F(\alpha)(r_g/r)^n$, neglecting quantities of higher order of smallness in r_g/r; the concrete form of the function $F(\alpha)$ would depend on the choice of the theory, but in any case $F(\alpha)$ would be on the order of unity (on the average in the interval 0, π). Consequently, the path difference $\Delta\Lambda$ between the polarization components of a beam reaching an observer on earth, in the case when the difference $\pi - \alpha$ is of the order of $\pi/2$ for the angle α seen by the observer (and particularly when $\alpha \ll 1$), can be estimated at

$$\Delta\Lambda \sim \rho(r_g/\rho)^n, \tag{2}$$

where ρ is the impact parameter of the beam.

If we substitute in (2) for r_g the sun's gravitational radius ($r_g \approx 3.0$ km), and for ρ the radius of the earth's orbit ($\rho \approx 1.5 \times 10^8$ km), we get $\Delta\Lambda \sim 6.0 \times 10^{-3}$ cm for n = 2. If we now take into account the fact that the angle α changes strongly during the year, owing the the earth's motion in its orbit, then it becomes obvious that the presence of gravitational birefringence even in the second order (n = 2) in G would be sufficient to cause fluctuations of the observed planes of the polarization of the light from nebulas and stars, referred to in Sec.1, and there would be no need to resort to observations during solar eclipses.

4. By way of an example of a theory that leads to gravitational birefringence, let us consider the electrodynamics of the linear tensor theory of gravitation (LTT), namely Eqs. (10a) of Moshinsky's paper [3]; the same equations are given in [4 - 6]. It turns out that

if the mixed components of the tensor gravitational potential used in these equations vanish identically[1], then these equations are equivalent to the ordinary Maxwell's equations for media at rest not only in the case considered by Moshinsky, when the gravitational potential is spatially isotropic, but also in the case of an arbitrary gravitational potential (provided the mixed components vanish). The corresponding dielectric and magnetic tensors $\varepsilon_{pq}^{LTT}(x)$ and $\mu_{pq}^{LTT}(x)$ are expressed as follows in terms of the components of the tensor gravitational potential (which is denoted as in [3]):

$$\epsilon_{pq}^{LTT} = (1 - h_{\rho\rho} - 2h_{44})\ \delta_{pq} - 2h_{pq},\ \mu_{pq}^{-1\,LTT} = (1 - h_{\rho\rho} + 2h_{44}) \times \times \delta_{pq} + 2h_{pq}, \quad (3)$$

where μ_{pq}^{-1} is the tensor inverse to μ_{pq}.

It is easy to see from (3) that the condition (1) will, in general, be violated (in second order in $h_{\mu\nu}$ and hence also in G) if h_{pq} is not proportional to δ_{pq}, i.e., it has a spatially-anisotropic form; but it is precisely an anisotropic tensor h_{pq} which describes in the LTT a spherically-symmetrical gravitation field in the case when the LTT leads to the same magnitudes for the three classical effects as the GRT (cf. [6], and also [4, 5])[2]. It is easy to verify that such a gravitational potential leads in the LTT to the same polarization effects as discussed in Secs. 1 and 3. If we consider, as in [1, 5], a variant of LTT leading (for spherically symmetrical fields) to a tensor h_{pq} of spatially-isotropic form, then the polarization effects arising as a result of this manner of violation of the equivalence principle are due not to the sun's gravitational field, but to the gravitational field of the revolving galaxy; however the method observing this effect differs from that indicated in Sec. 1.

In conclusion, the author is grateful to Ya. A. Smorodinskii for help and I. D. Novikov for a discussion.

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[1] Precisely such gravitational potentials were considered in [3 - 6].

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E R R A T U M

Article by O. N. Naida, Vol. 8, No. 2.

On page 69, lines 5 - 6 from top, read "... necessary and sufficient condition for the absence of birefringence" instead of "...necessary and sufficient condition for birefringence."