FLUX QUANTIZATION IN A NORMAL METAL

I.O. Kulik
Physico-technical Institute of Low Temperatures, Ukrainian Academy of Sciences
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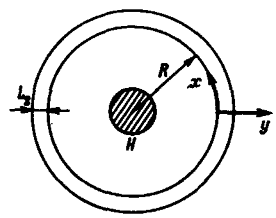
The effect of flux quantization in hollow thin-wall superconducting cylinders [1 - 4] is connected with the occurrence of a circulating current J or, what is the same, of a magnetic moment M = JS/c (S is the cross section of the cylinder), varying periodically as a function of the magnetic field flux ϕ. In a preceding paper [5], the author has shown that a similar phenomenon takes place also in the normal state of the superconductor at a temperature $T > T_c$ when there is no long-range order. In the latter case, the effect is due to "fluctuation pairing" of the electrons [6]. It will be shown in this paper that the quantization of the flux is not connected with the presence of superconducting long-range order (of the ODLRO type [3]) and can occur under conditions when the quantum size effect [7] appears in a perfectly normal metal as a consequence of the sensitivity of the quantum states of the electron to the field of the vector potential A (the so-called Aharonov-Bohm effect [8]).



Let us consider a hollow thin-wall metallic cylinder (see the figure) placed in a field with a vector potential A, produced by a "pointlike" source of magnetic field (shown shaded in the figure), namely a narrow solenoid, the stray field of which can be neglected. The coordinate x is measured along the perimeter of the ring (length of the perimeter $L_1 = 2\pi R$), y along the radius (wall thickness L_2), and z along the cylinder axis. Assuming L_2 to be small, we can consider the vector potential in the region occupied by the electrons to be constant, $A = A_x = \phi/L$, where ϕ is the flux produced by the solenoid. Solving Schrodinger's equation in the region shown in the figure, we readily obtain the wave functions and the energy levels of the electron

$$\psi_{nmq} = \mathrm{const}\; e^{ik_n x} e^{iqz} \sin\frac{\pi m y}{L_2}, \quad k_n = \frac{2\pi n}{L_1}, \tag{1}$$

$$E_{nmq} = \frac{\hbar^2}{2m^*}\left[\left(k_n - \frac{eA}{\hbar c}\right)^2 + q^2 + \frac{\pi^2 m^2}{L_2^2}\right]. \tag{2}$$

Using the expression (2) for the spectrum, we can easily calculate the free energy F of the system

$$F = N\zeta - T\sum_{n=-\infty}^{\infty}\sum_{m=1}^{\infty}\int_{-\infty}^{\infty}\frac{L_3\,dq}{\pi}\ln\left(1 + e^{\frac{\zeta - E_{nmq}}{T}}\right). \tag{3}$$

We shall consider for simplicity the case of sufficiently small thicknesses L_2, when the quantum limit with respect to m is satisfied, so that it is possible to take into account only the lowest level m = 1. From the condition of normalization to the total number of particles, the chemical potential is

$$\zeta - \frac{\pi^2\hbar^2}{2m^* L_2^2} = \zeta_0 = \frac{\pi\hbar^2}{m^*} N L_2$$

(N is the electron density).

According to (2) and (3), the free energy depends on the total flux $\phi = L_1 A$.

Differentiating F with respect to ϕ, we obtain the magnetic moment of the system μ ($\mu = M/S$):

$$\mu = -\partial F/\partial \phi, \tag{4}$$

which thus turns out to be different from zero. It is easy to see that μ oscillates as a function of the flux, with a period equal to the quantum $\phi_0 = hc/e$, which, naturally contains the single electron charge e. Using (2) - (4), we obtain with the aid of the Poisson formula

$$\mu = \sum_{p=1}^{\infty} \mu_p \sin\left(2\pi p \frac{\phi}{\phi_0}\right) , \tag{5}$$

where the coefficients μ_p are determined by the following formulas:

a) for degenerate statistics ($\zeta_0 >> T$)

$$\mu_p = \frac{2eTk_0L_3}{\hbar c}\left(\frac{2}{\pi p L_1 k_0}\right)^{1/2} \frac{\sin(pL_1k_0 - \pi/4)}{\mathrm{sh}\left(p\frac{\pi m^* L_1}{\hbar^2 k_0} T\right)}, \quad k_0 = \left(\frac{2m^*\zeta}{\hbar^2}\right)^{1/2}; \tag{6}$$

b) for nondegenerate statistics ($\zeta_0 << T$)

$$\mu_p = -\frac{Neh}{m^* c R} L_2L_3 \frac{pT}{\epsilon_0} e^{-\pi^2p^2T/\epsilon_0}, \quad \epsilon_0 = \frac{\hbar^2}{2mR^2}. \tag{7}$$

Thus, owing to the quantization of the electron motion in the ring and to the sensitivity of the quantum states to the vector potential $\vec{A}$, a magnetic moment μ appears and oscillates as a function of the flux ϕ. The presence of such a moment is equivalent to the existence of a circular current in the ring, but this is not the ordinary conduction current, but a "diamagnetic" current analogous to that introduced for the interpretation of Landau diamagnetism (cf., e.g., [9]). Unlike superconductors, where the quantization of the flux is connected with the cooperative motion of Cooper pairs, there is no long-range order in this case. The motion of the individual electrons is independent, and the collisions can cause the electrons to become redistributed among the states, but the average current remains different from zero as a consequence of the dependence of the energies of the individual states, and hence of the total energy, on A. The current state corresponds in this case to a minimum of the free energy, so that allowance for dissipation does not lead to its decay.

Just as in the calculation of the oscillating part of the Landau diamagnetism, the role of scattering of the electron reduces in the present problem to a smearing of the energy levels, and consequently to a decrease of the amplitude of the oscillating terms in (5). Phenomenologically this can be taken into account by introducing the "Dingle factor" $\exp(-\hbar/\tau\Delta E)$, where ΔE is the distance between levels and τ is the free path time (which also takes into account the diffuseness upon reflection from the walls). Summarizing, we can conclude that the effect can be observed at sufficiently low temperatures ($T << \Delta E \sim \hbar^2k_0/m^*L_1$) large mean free paths ($\ell \gtrsim L_1 = 2\pi R$), and high specularity of the reflection ($1 - p \lesssim L_2/L_1$). It is difficult to satisfy these conditions in experiments, even in the case of metals such as bismuth[1)].

[1)]Observation of the effect is not necessarily predicated on a direct measurement of M. Thus, the longitudinal conductivity of the cylinder is an oscillating function of the flux in the presence of a homogeneous field. This is analogous to the Parks-Little effect [10], namely oscillations of the resistance on the transition curves of a superconducting cylinder in a field.

We note that none of the foregoing contradicts the general theorems concerning the connection between flux quantization and the presence of superconducting long-range order [3]. Formally, this effect is not a macroscopic quantum effect, since in a large system (as $R \to \infty$) the magnetic moment μ vanishes (see (6) and (7)). It is more readily analogous to the diamagnetism of closed organic molecules. However, as shown by the foregoing estimates, when certain conditions are satisfied, the effect can appear in samples of rather large dimensions, usually regarded as "macroscopic."

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NONLINEAR WAVES IN A RELATIVISTIC ELECTRON BEAM

Ya.B. Fainberg, V.D. Shapiro, and V.I. Shevchenko
Physico-technical Institute, Ukrainian Academy of Sciences
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It is known that in stationary electronic configuration (the self-stabilized Budker beam [1], Veksler rings [2]), a decrease of the Coulomb repulsion pulses is attained with the aid of the Lorentz forces due to the contraction of the currents of relativistic electrons. Since the limitation of the wave amplitude in a plasma is connected with the action of Coulomb repulsion of the electrons, the natural question arises whether it is possible to excite large-amplitude currents by enhancing the foregoing compensation. Such a situation obtains when wave propagate along the axis of an electron beam whose particles rotate azimuthally (a system of the E-layer type [3]). In this case the charge-density wave leads to oscillations of the particle current in the beam $j'_\phi(z - v_{ph}t) = -ev_0 n'(z - v_{ph}t)$ (v_0 is the azimuthal velocity of the beam), and consequently to the appearance of the magnetic field of the wave, $H_r(z - v_{ph}t)$. the self-contraction force, produced by this magnetic field in the plasmoids into which the wave breaks up the beam, $F_H = ev_0H_r/c$, just as in the stationary case, is in anti-phase with the Coulomb force $F_E = -eE_z$, and leads at $v_0 \approx (c^2 - v_{ph}^2)^{1/2}$ to an appreciable decrease of the electron displacement in the field of the wave. A wave can then propagate in the beam, having a very large electric-field amplitude, without intersection of the trajectories and breaking of the wave front. This result points to the possibility of effectively using waves in relativistic beams to implement the plasma method of acceleration proposed in [4].