[8] A.P. Levanyuk, Fiz. Tverd. Tela 5, 1776 (1963) [Sov. Phys.-Solid State 5, 1294 (1964)].
[9] J. Villain, Sol. State Comm. 8, 295 (1970).

EXCHANGE INTERACTION OF AN ISOBAR WITH A NUCLEON, AND DYNAMIC MODEL OF TWO-BARYON RESONANCE

V.A. Karmanov, L.A. Kondratyuk, and I.S. Shapiro
Institute of Theoretical and Experimental Physics, USSR Academy of Sciences
Submitted 24 April 1970
ZhETF Pis. Red. 11, No. 11, 543 - 547 (5 June 1970)

This paper is devoted to multibaryon resonances, or nuclei in which one of the nucleons is replaced by a baryon resonance. The possible existence of such systems was suggested in [1, 2]. In [3], the general premises of the theory of resonance-nuclei were elucidated and the nuclei (ΔN) and (Δ2N) were considered (Δ - isobar with quantum number $T = 3/2$, $I^P = 3/2^+$, $m_\Delta = 1236$ MeV). On the basis of an analysis of the experimental data and phase-shift data, it was shown that the two-baryon resonance (ΔN) with isospin $T = 1$ makes a small contribution to the cross sections of the reactions $\pi d \to \pi NN$, $\pi d \to NN$, and $NN \to NN$. It was also established that induced non-exchange interaction of ΔN is not strong enough to form a bound state. In the present article we investigate the exchange forces due to the existence of the decay interaction $\Delta \to \pi N$ (see Fig. 1).

Our starting point is the Lippmann-Schwinger integral equation (formula (16) of [3]), the kernel of which is in this case the amplitude corresponding to the diagram (Fig. 1). If the kinetic energies of the baryons are small compared with the mass differences of the isobar and the nucleon $m_\Delta = 300$ MeV, then this amplitude can be regarded as dependent only on the momentum transfer $\vec{q}$, and the dependence of the isobar width Γ on the kinematic variables can be completely neglected. Then the aforementioned equation in the coordinate representation goes over into the Schrodinger equation with local energy-independent complex potential. The role of the energy in this equation is played by the quantity $E' = E + \Gamma/2$, where E is the total kinetic energy of the isobar and of the nucleon, and the operator of the potential is given by

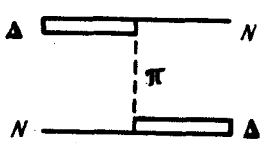

Fig. 1

$$\hat{V}(r) = \left(\frac{1}{2} + \frac{2}{3}\vec{\tau}_\Delta \vec{\tau}_N\right)\left\{-\left(\frac{1}{2} + \frac{2}{3} S\vec{\sigma}\right)\left(V_0(r) + \frac{5}{4}V_2(r)\right) + \left[2(S n)^2 S\vec{\sigma} + 2i(S n)S[\vec{\sigma}\times n] + \frac{1}{2}(S n)^2 - 3 S n \vec{\sigma} n\right] V_2(r)\right\}, \tag{1}$$

where S, $\vec{\sigma}$ and $\vec{\tau}_\Delta$, $\vec{\tau}_N$ are the spin and isospin operators of the isobar and of the nucleon, and

$$V_\Lambda(r) = -\frac{\lambda^2}{4\pi^2\mu^2}\int_0^\infty \frac{q^4 dq}{q^2 + \mu^2 - (\Delta m - i\Gamma/2)^2}\, i^\Lambda j_\Lambda(qr)\, g^2(q^2). \tag{2}$$

Here μ is the pion mass, $\Gamma = 120$ MeV, and $\lambda = 2$. The form factor $g(q^2)$, which takes into account the departure of the pion and the nucleon from the mass shell at the vertex ΔπN, is chosen in the form $g(q^2) = (q_0^2 + c^2)/(q^2 + c^2)$, where $q_0 = 231$ MeV/c. From the data on the production of the Δ isobar in πN and NN collisions, it can be concluded (see, e.g., [4]), that the constant c is close to 3μ. Bearing in mind the possible uncertainty in the value of c, we have investigated the sensitivity of the results to variations of c.

Figure 2 shows plots of the potentials $V_0(r)$ and $V_2(r)$. Curves 1, 2, and 3 correspond to the potentials Re $V_0(r)$, Im $V_0(r)$, and Re $V_2(r)$ at $c = 3\mu$. The dashed curve shows Re $V_0(r)$ at $c = 2\mu$. For comparison, the same figure (curve 4) shows the centrifugal potential in the p-state. The repulsion produced by it is larger than the attraction due to the exchange forces. This means that the only states that can be bound (by bound are meant states whose energy E' is less than zero) are s-states with quantum numbers $T = 2$, $I^P = 1^+$, and $T = 1$, $I^P = 2^+$. In this respect, the present results do not agree with those of [5]. In the latter, on the basis of a numerical solution of the Faddeev equations for the πNN system, it is stated that the strongest attraction is realized in the states 0^+ and 2^-. In analyzing the s-state, we shall neglect the potential V_2. This potential intermixes the s- and d-states, but the wave functions in the d-state are small in the region where an important role is played by the potentials V_0 and V_2, owing to the large centrifugal barrier. In addition, the potential V_2 has a certain numerical small factor compared with the potential V_0. If we neglect the potential V_2, then the potentials for states $T = 2$, $I^P = 1^+$ and $T = 1$, $I^P = 2^+$, the potentials turn out to be equal to $V_{21} = V_{12} = V_0/3$.

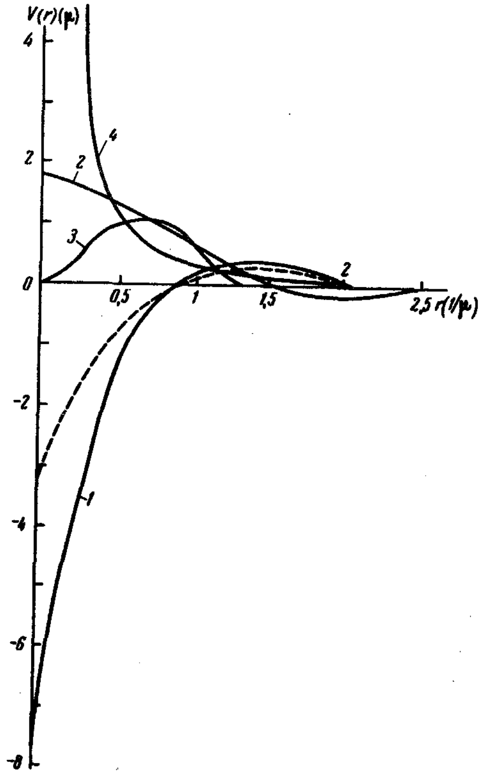

Fig. 2

As is well known, the upper bound of the number of s-levels in a monotonic attracting potential Re V_{12} is given by the formula [6]

$$n \leqslant I \text{ where } I = \frac{2}{\pi} \int_0^\infty (2m|\mathrm{Re}\, V_{12}|)^{1/2} dr. \tag{3}$$

At $c = 2\mu$ and 3μ, the integral I is equal to 0.95 and 1.25, respectively. To estimate the binding energy, we approximate the potential by a triangular well: Re $V_{12} = -b(r_0 - r)$ for $r \leq r_0$ and Re $V_{12} = 0$ for $r \geq r_0$. We set the width of the well equal to $0.8\ \mu^{-1}$, and the slope is determined from the condition that I be constant. Solving the Schrodinger equation $\chi'' = (\mathrm{Re}\, V_{12} + \kappa^2)\chi = 0$, where $\kappa^2 = -2mE'$, we get $\kappa = 0.1\mu$ at $c = 3\mu$ and $\kappa = -0.15\mu$ at $c = 2\mu$. The real part of the level energy is thus close to zero in both cases.

Allowance for the imaginary part of the potential makes κ complex and causes the position of the pole of the amplitude with respect to E' to be given by the formula

$$E' = C_1 + iC_2. \tag{4}$$

The sign of C_2 is determined by the nature of the level and by the sign of the imaginary part of the potential. In our case Im $V_{12} > 0$, and therefore for a real level ($\kappa > 0$) and for a resonance ($C_1 > 0$) C_2 is positive, and for a virtual level($\kappa < 0$) it is negative[1]).

In order to estimate the values of the constants C_1 and C_2, let us consider the solution of the Schrodinger equation in a complex well $(-2mVr_0^2)^{1/2} = \alpha - i\beta$, setting α equal to $\pi/2$, corresponding to the solution $\kappa = 0$ at $\beta = 0$. It turns out that when the ratio β/α changes in the interval $0 < \beta/\alpha < 0.3$, the constants C_1 and C_2 change in the intervals $0 < C_1 < 10$ MeV and $0 < C_2 < 10$ MeV. Calculation shows that the uncertainties in the parameters of the potential, connected with neglecting the kinetic energies of the varriers, can likewise not change the values of C_1 and C_2 by more than 10 MeV.

Thus, the strongest attraction between the isobar and the nucleon as a result of N exchange interaction occurs in states with quantum numbers T = 2, $I^P = 1^+$, and T = 1, $I^P = 2^+$. This attraction can lead to the formation of either a weakly-bound state (real or virtual) or a resonance. The bound state or the resonance in the ΔN system becomes manifest as a two-baryon resonance, and the antibound state is manifest at a certain definite structure with a distribution with respect to the πNN mass which, generally speaking, is weaker than in the case of a bound state or a resonance. Since these singularities are at a distance Γ/2 from the physical region, the three indicated possibilities can be distinguished experimentally only when the value of C_1 is comparable with Γ/2. We must bear in mind one more important circumstance that hinders greatly the search for such resonances. The pole of the amplitude with respect to E', corresponding to two-baryon resonance, is always close to the cut with respect to E'. The distance to the start of the cut $(C_1^2 + C_2^2)^{1/2}$ is in this case even smaller than the distance $\Gamma/2 - C_2$ to the physical region. The existing experimental information concerning the peaks in two-nuclear systems could be satisfactorily described up to now only by taking into account just the non-pole singularities (see, e.g., [8 - 9]). In order to separate the poles corresponding to the resonances, it is necessary to study, besides the dependence on E, the variation of the position and of the form of the resonance peak as a function of the values of other kinematic invariants, for example, the momentum transfer (see [10]).

The authors are grateful to A.B. Kaidalov and A.E. Kudryavtsev for a discussion of a number of questions touched upon in the work.

[1] V.G. Grishin and M.I. Podgoretskii, JINR Preprint R-1508, 1964.
[2] G.A. Leksin, Proceedings of the Second Winter School on High-energy Physics and Nuclear Theory, Leningrad, 1968.
[3] L.A. Kondratyuk and I.S. Shapiro, Yad. Fiz. 12 (1970) [Sov. J. Nucl. Phys. 12 (1970)].
[4] E. Ferrari,and F. Selleri, Nuovo Cim. 27, 1450 (1963); G. Wolf, Phys. Rev. 182, 1538 (1969).
[5] W.A. Gale and J.M. Duck, Nucl. Phys. B8, 109 (1968).
[6] F. Calogero, Variable Phase Approach to Potential Scattering, Academic Press, New York and London, 1967.
[7] D.D. Brayshow and R.F. Peierls, Phys. Ref. 177, 2539 (1969).
[8] O.D. Dal'karov, Yad. Fiz. 7, 820 (1968) [Sov. J. Nucl. Phys. 7, 499 (1968)].
[9] D. Evrard, A. Fridman, and A.C. Hirshfeld, Nucl. Phys. B14, 699 (1969).
[10] O.D. Dal'karov and I.S. Shapiro, Phys. Lett. 26B, 706 (1968); Yad. Fiz. 7, 562 (1968) [Sov. J. Nucl. Phys. 7, 349 (1968)].

[1])The fact that the resonance can be situated on the first sheath of E' does not contradict the unitarity condition, since this part of the first sheath of E', for which Im E' < Γ/2, is itself in the second unphysical sheath of the energy W of the πNN three-particle system (see, e.g., [7]). C_2 is always smaller than Γ/2, and the W pole can obviously not go off to the physical sheath.