3) If the weak turbulence is "quasirandom," then there is practically neither diffusion nor heating. 4) Our analysis does not apply to the results of quasilinear theory and the theory of strong turbulence.

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FEATURES OF ELECTRON DRAG OF DISLOCATIONS IN SUPERCONDUCTORS

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Several experimental groups have recently observed an influence of the superconducting transition on the kinetics of plastic deformation of metals [1 - 3][1]. Since the properties of the crystal lattice remain practically unchanged in a superconducting transition, it is natural to ascribe the observed effect to the influence of conduction electrons. The possible mechanism of such an influence is as follows. As is well known, the plastic properties of crystal are determined in final analysis by the mobilities of the individual dislocations. Kravchenko has shown earlier [4] that a dislocation moving in a normal metal experiences a drag force caused by its interaction with the conduction electrons. This force has an appreciable magnitude and its variation, expected in the case of the superconducting transition, can greatly influence the balance of forces that determine the dislocation mobility. In this communication we present results of an investigation of the force of electron drag by dislocations in superconductors. It will be shown below that when the metal goes over from the normal to the superconducting state the character of the variation of this force is not as simple as assumed earlier [4, 5], but reveals a number of interesting features.

A dislocation moving with constant velocity $\vec{V}$ produces in a crystal an alternating field of elastic deformations $u_{in}(\vec{r} - \vec{V}t)$, which exerts on the conduction electron a force determined by the deformation potential $\lambda_{in}u_{in}(\vec{r} - \vec{V}t)$ (the components of the tensor λ_{in} are of the order of the Fermi energy ε_F [6]). Because of this potential, a moving dislocation produces transitions in the electron system and loses its energy to the perturbation of this system. The dislocation retardation force due to such losses is equal in absolute magnitude to the energy absorbed by the electrons when the dislocation moves on a unit path.

It can be shown that the Hamiltonian of the interaction between the electrons and the moving dislocation has in the second-quantization representation the form

[1]The cited articles are not the only publications in which the influence of the superconducting transition on plastic properties of metals is noted. We have cited only the investigations in which, in our opinion, the effect is most clearly pronounced.

$$\mathcal{H}_{\text{int}} = \frac{1}{L_1 L_2} \sum_q \sum_k \lambda_{in} u_{in}^q e^{-i\omega_q t} (a^+_{k+q\uparrow} a_{k\uparrow} + a^+_{k+q\downarrow} a_{k\downarrow}),$$

$$\omega_q = qV. \tag{1}$$

Here u_{in}^q is the Fourier component of the deformation field, the wave vector q lies in the plane perpendicular to the dislocation axis, L_1 and L_2 are the dimensions of the crystal in this plane, $a^+_{k\uparrow}$ and $a_{k\uparrow}$ are the operators of creation and annihilation of electrons with appropriate spin direction. Changing over in the standard manner to the elementary-excitation operators $\gamma^+_{k\uparrow}$ and and $\gamma_{k\uparrow}$ of the superconductor (we use the notation of [7]), we obtain

$$\mathcal{H}_{\text{int}} = \frac{1}{L_1 L_2} \sum_q \sum_k \lambda_{in} u_{in}^q e^{-i\omega_q t} [(u_{k+q} u_k - v_{k+q} v_k)(\gamma^+_{k+q\uparrow}\gamma_{k\uparrow} + \gamma^+_{k+q\downarrow}\gamma_{k\downarrow}) + (u_{k+q} v_k + v_{k+q} u_k)(\gamma^+_{k+q\downarrow}\gamma^+_{-k\uparrow} + \gamma_{-k-q\uparrow}\gamma_{k\downarrow})]. \tag{2}$$

The largest contribution to the retardation force is made by transitions caused by waves with the maximum values $q \sim q_m$; $q_m \sim 1/r_0$ if $1/r_0 < 2k_F$, or $q_m \sim 2k_F$ in the opposite case, r_0 is the radius of the dislocation nucleus, and $2\hbar k_F$ is the diameter of the Fermi surface. Since r_0, $1/2k_F \sim a \ll \ell$ (ℓ is the mean free path of the electrons, a is the lattice constant), it follows that the energy absorbed by the electrons can be calculated by assuming equilibrium occupation numbers of the elementary excitations [6, 8, 9]. The retardation force per unit length of dislocation F(V) is defined by the expression

$$F(V) = \frac{1}{L_3 V} \sum_q \sum_k \hbar\omega_q \nu_{k,k+q}. \tag{3}$$

Here L_3 is the dislocation length, and $\nu_{kk'}$ is the frequency of the transitions with absorption of energy $\hbar\omega_{k'-k}$, calculated by perturbation theory with the matrix elements of the Hamiltonian (2) and with occupation numbers $f(\varepsilon_k) = [1 + \exp(\varepsilon_k/T)]^{-1}$:

$$\nu_{kk'} = \frac{2\pi}{\hbar}\left|\frac{\lambda_{in} u_{in}^{k'-k}}{L_1 L_2}\right|^2 \Big\{ 2(u_{k'}u_k - v_{k'}v_k)^2 [f(\varepsilon_k) - f(\varepsilon_{k'})]\delta(\varepsilon_{k'} - \varepsilon_k - \hbar\omega_{k'-k}) + (u_{k'}v_k + v_{k'}u_k)^2 [1 - f(\varepsilon_k) - f(\varepsilon_{k'})][\delta(\varepsilon_{k'} + \varepsilon_k - \hbar\omega_{k'-k}) - \delta(\varepsilon_{k'} + \varepsilon_k + \hbar\omega_{k'-k})]\Big\}. \tag{4}$$

We confine ourselves henceforth, for simplicity, to the case of screw dislocation, for which

$$\lambda_{in} u_{in}^q = ib \frac{\lambda_{13} q_2 - \lambda_{23} q_1}{q^2}, \tag{5}$$

if the "3" axis is directed along the velocity V of the dislocation; b is the magnitude of the Burgers vector.

Substituting (4) and (5) in (3), changing over from summation to integration, and recognizing that waves with $q \ll k_F$ make an insignificant contribution to the drag force F(V), we obtain:

$$F(V) = \frac{m^2 b^2 \lambda^2}{3\pi^3 \hbar^4} \left\{ 2 \int_0^{\hbar q_m V} \frac{d\epsilon'}{\epsilon'} \int_\Delta^\infty d\epsilon \frac{\epsilon(\epsilon'+\epsilon) - \Delta^2}{\sqrt{\epsilon^2 - \Delta^2}\sqrt{(\epsilon'+\epsilon)^2 - \Delta^2}} \times \right.$$

$$\times [f(\epsilon) - f(\epsilon'+\epsilon)] + \theta(\hbar q_m V - 2\Delta) \int_{2\Delta}^{\hbar q_m V} \frac{d\epsilon'}{\epsilon'} \int_\Delta^{\epsilon'-\Delta} d\epsilon \frac{\epsilon(\epsilon'-\epsilon) + \Delta^2}{\sqrt{\epsilon^2 - \Delta^2}\sqrt{(\epsilon'-\epsilon)^2 - \Delta^2}} \times$$

$$\left. \times [1 - f(\epsilon) - f(\epsilon'-\epsilon)] \right\}, \tag{6}$$

$$\theta(x) = \begin{cases} 1, & x > 0, \\ 0, & x < 0, \end{cases}$$

where $\lambda^2 = \lambda_{13}^2 + 2\lambda_{23}^2$, and m is the electron mass.

Without dwelling here on a detailed analysis of (6), we write out expressions for F(V) in a number of limiting cases.

At absolute zero temperature ($T = 0$, $\Delta = \Delta_0$)

$$F(V) = \theta(\hbar q_m V - 2\Delta_0) \frac{m^2 b^2 \lambda^2}{3\pi^3 \hbar^4} \int_{2\Delta_0}^{\hbar q_m V} \frac{d\epsilon'}{\epsilon'} \int_{\Delta_0}^{\epsilon'-\Delta_0} d\epsilon \frac{\epsilon(\epsilon'-\epsilon) + \Delta_0^2}{\sqrt{\epsilon^2 - \Delta^2}\sqrt{(\epsilon'-\epsilon)^2 - \Delta^2}}. \tag{7}$$

We see therefore that the force of the electron retardation is equal to zero at dislocation velocities $V < V_{cr} = 2\Delta_0/\hbar q_m$. When $V > V_{cr}$, the drag force differs from zero, and if $V - V_{cr} \ll V_{cr}$, we have in order of magnitude

$$F(V) \approx \frac{n \epsilon_F b^2 q_m}{4 v_F} (V - V_{cr}), \tag{8}$$

where n is the density of the electrons in a normal metal and v_F is the Fermi velocity. The appearance of the critical velocity is the consequence of the existence of a gap in the energy spectrum of the superconductor, and has the same nature as threshold absorption of the electromagnetic [10] and acoustic [8] energy by a superconductor. The critical velocity is of the order of $v_F/\rho q_m$, where ρ is the radius of the Cooper pair ($\rho \sim 10^{-4}$ cm), i.e., it is smaller by one order of magnitude than the velocity of sound, and is therefore attainable experimentally. At velocities larger in comparison with V_{cr}, the electron drag force in the superconductor is of the same order as that calculated in [4] for a normal metal.

If $T \neq 0$, but $T \ll \Delta$ and $V < V_{cr}$, we have

$$F(V) \approx \begin{cases} \dfrac{n\epsilon_F b^2 q_m}{\pi v_F} e^{-\Delta/T} V, & \hbar q_m V \ll T \ll \Delta \\[2ex] \dfrac{n\epsilon_F b^2}{\pi\hbar v_F} \sqrt{T}\, e^{-\Delta/T} \sqrt{\hbar q_m V}, & 2\Delta > \hbar q_m V \gg T \end{cases} . \qquad (9)$$

Attention must be called to the fact that F(V) becomes nonlinear at sufficiently high velocities.

Formulas (6) - (9) show that the force of the electron drag by dislocations in a superconductor has a complicated dependence on the velocity and on the temperature.

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CONTRIBUTION TO THE THEORY OF THE MIXED STATE OF SUPERCONDUCTORS OF THE SECOND KIND

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A method is proposed for describing a superconductor in the case when the Cooper pairs can form several condensates, i.e., they can accumulate in several different states. This problem is meaningful in the presence of degeneracy of the ground state of the Cooper pair, as is the case in a superconductor of the second kind in a magnetic field.

The starting point is the Ginzburg-Landau theory [1]. In this theory, the free energy is