# DEPENDENCE OF THE CRITICAL CURRENT ON THE DISLOCATION DENSITY

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Much attention has been paid recently in the literature to the interaction between Abrikosov vortices and elastic inhomogeneities of the crystal lattice [1 - 5]. The pinning of vortex filaments and hence the possibility of obtaining large currents in the superconductors of the second kind in the mixed state is usually attributed to this interaction. The main role in the pinning of the vortices is played by inhomogeneities of the elastic fields of the dislocations.

The interaction of a single vortex with one dislocation was considered in [1 - 3], and the pinning force was estimated on the basis of the expression derived for the interaction energy.

However, the result obtained in this manner may turn out to be incorrect, for at the high dislocation densities ($10^{10}$ $cm^{-2}$) for which the maximum pinning force is expected according to [1], the elastic-field distribution may be greatly distorted even near the dislocation.

We obtain in this paper the binding energy of the vortex as a function of the dislocation density, assuming the dislocations to have a homogeneous and isotropic distribution, on the basis of a simple model of the interaction between the vortex and the elastic fields, with account taken only of the so-called volume factor.

We represent the vortex in the form of a cylindrical region with a homogeneous spontaneous dilatation, equal to the relative change $\Delta v$ of the unit-cell volume during the phase transition. Then the interaction energy of a vortex of unit length with the elastic fields is

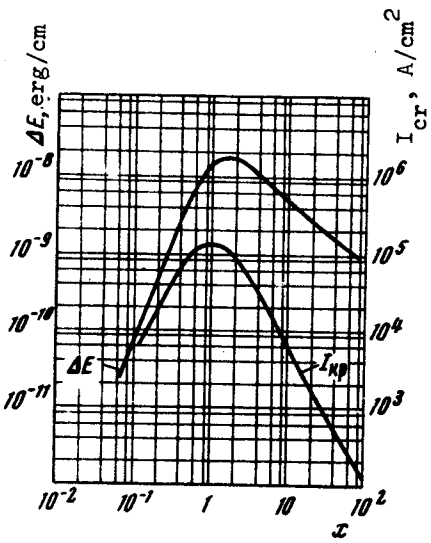
$$E \approx \Delta v\, \xi^2 K \bar{\epsilon}, \tag{1}$$

where $\xi$ is the radius of the vortex, K the bulk modulus, and $\bar{\epsilon}$ the average dislocation-induced dilatation of the lattice in the region occupied by the vortex. The order of magnitude of the vortex binding energy can be determined as the difference between the possible maximal and minimal values of this energy.

Following [6], we define a function $I(\varepsilon)$ representing the probability density of different values of the dilatation or, which is the same, the relative number of cells with a specified dilatation. The approximate form of this function is

$$I(\epsilon) = \frac{1}{\sqrt{2\pi}\,\epsilon_0} \exp\left(-\frac{\epsilon^2}{2\epsilon_0^2}\right), \tag{2}$$

where $\varepsilon_0 = b\sqrt{\rho}$, b is the magnitude of the Burgers vector, and $\rho$ is the density of the edge dislocations. Although (2) is valid for deformations in the interval $10^{-2} \geq |\varepsilon| \geq 10^{-6}$, we shall use it in the entire interval, assuming that the error in the final result will be negligible if $\bar{\varepsilon}$ lies in the indicated interval.




Since the quantity $d \sim \rho^{-1/2}$ is the parameter of the inhomogeneity of the dilatational field of the crystal, it can be assumed that the minimum volume in which the relative number of cells with a given dilatation is still given by (2), if the parameter is suitably chosen, is of the order of $d^3$. Inside the vortex, this number is not always given by (2). However, for any vortex location it is always possible to add to the vortex a volume on the order of $d^3$ such that the dilatation distribution in the obtained volume is given by (2). Thus, if it is assumed that the mobility of the vortex is determined by a mobility of a vortex section of length d, then the average dilatation in the vortex of this length will be determined by the manner in which $n = (\xi^2/\Omega)d$ cells contained in a vortex of length d will fill $N = [(\xi^2 + d^2)/\Omega]d$ "states" with dilatations distributed in accordance with (2) ($\Omega$ is the volume of the unit cell).

$\bar{\varepsilon}$ will have the maximum negative value if n cells fill all the "states" completely, starting with the largest negative values of the dilatations. There is no such possibility, however, since the vortex, being a compact formation, will overlap also regions with large dilatations. We shall therefore assume that when the minimum average dilatation is determined the probability of filling the "states" with dilatation $\varepsilon$ can be specified by the distribution function of the random quantity

$$F(\epsilon) = \frac{1}{2}\left[1 - \phi\left(\frac{\epsilon - \epsilon_1}{\theta}\right)\right], \tag{3}$$

where $\phi(z)$ is the probability integral, $\varepsilon_1$ is the state, still to be determined, with a filling probability 1/2, and $\theta$ is a characteristic width that takes into account the aforementioned overlap of the regions with large dilatations,

$$\theta = \epsilon_0\, \xi^2/d^2,$$

where $\varepsilon_0$ is the normal width characterizing the distribution (2), and the ratio $\xi^2/d^2$ is the measure of the overlap.

The quantity $\varepsilon_1$ is determined from the equation

$$\frac{n}{N} = \int_{-\infty}^{+\infty} l(\epsilon)F(\epsilon)\,d\epsilon = \frac{1}{2}\left[1 - \phi\left(\frac{\epsilon_1}{\sqrt{\epsilon_0^2 + \theta^2}}\right)\right], \tag{4}$$

and the minimal average dilatation in the vortex is given by

$$|\bar{\epsilon}_{min}| = \frac{N}{n}\int_{-\infty}^{+\infty} l(\epsilon)F(\epsilon)\,\epsilon\,d\epsilon = \frac{N\epsilon_0^2}{\sqrt{2\pi}\,n\sqrt{\epsilon_0^2 + \theta^2}}\exp\left(-\frac{\epsilon_1^2}{2(\epsilon_0^2 + \theta^2)}\right). \tag{5}$$

Substituting (5) in (1) we obtain for the binding energy of the vortex

$$\Delta E = \Delta v K b\, \xi\, \frac{x(1 + x^2)}{\sqrt{1 + x^4}} \exp\left(-\frac{1}{2}u^2\right), \tag{6}$$

where u is determined from the equation

$$\phi(u) = 1 - \frac{2}{1 + x^2},$$

and $x = d/\xi$.

The dependence of $\Delta E$ on x is shown in the figure. The maximum value of the binding energy reached at $x = 1.7$, corresponding at $\xi \simeq 10^{-5}$ cm to a dislocation density $\rho \simeq 3 \times 10^9$ cm$^{-2}$, is of the order of $10^{-8}$ erg/cm (we used in the estimate the values $\Delta v = 3 \times 10^{-7}$ [2] and $K = 10$ dyne/cm$^2$). With increasing dislocation density, the binding energy tends rapidly to zero. It also decreases with decreasing dislocation density.

If we use for the estimate of the pinning force the approximation $F = \Delta E/d$, then we obtain $J_{cr} \simeq 10^5$ A/cm$^2$ for the density of the critical current at the maximum binding energy. The figure shows also the dependence of the density of the critical current on the dimensionless parameter x.

It should be noted that the foregoing results are valid only for a uniform dislocation distribution. If the dislocations are not uniformly distributed, then the large values of the critical-current density can be associated with the existence of dislocation walls [7], in which large dislocation densities can be reached also at relatively low average density.

The relation obtained by us is expected to hold for strongly deformed samples in which the dislocation distribution is close to homogeneous.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