If follows from the foregoing results that, just as Cerenkov radiation of an individual beam particle causes oscillations to build up, the radiation of individual particles passing through an inhomogeneous medium or having non-uniform motion leads to the development of collective instabilities of the beam. The growth increment of the waves depends in this case both on the emissivities of the individual particles ($\partial\varepsilon/\partial z$ or $\partial v_0/\partial t$) and on the collective characteristics of the beam (ω_b).

In conclusion, the author thanks Ya.B. Fainberg and V.I. Kurilko for interest in the work and for useful discussions.

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DRAGGING OF A LIQUID BY A LIQUID THROUGH A STATIONARY SOLID WALL

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The influence of acoustic fluctuations (phonons) on the hydrodynamic properties of a liquid was considered in [1]. An account of these fluctuations is of particular importance in those cases when one deals with phenomena that might exist in principle, but are absent in ordinary hydrodynamics. For example, the thermomechanical effect, which arises only when phonons are taken into account [1], does not exist in ordinary hydrodynamics.

The purpose of the present article is to call attention to the following effect, which is possibly the most pronounced manifestation of phonons. We consider two liquid layers separated by a stationary solid partition (Fig. 1). Assume that Poiseuille flow takes place in region I. According to the equations of ordinary hydrodynamics, the liquid in region II remains stationary. The situation is altered if account is taken of the possibility of momentum transfer from region I into region II by phonons passing through the solid partition. This should be accompanied by dragging of the liquid in region II, so that the velocity distribution should have the form shown in Fig. 1.

The drag velocity can be calculated on the basis of the equations of motion derived in [1]. We write first the kinetic equation for the phonons in the liquid

$$\frac{\partial \chi}{\partial z} + \frac{\chi}{c n_z \tau} = \frac{\varepsilon n_x}{c} \frac{dv}{dz}, \qquad (1)$$

where the function χ determines the deviation of the phonon distribution function from the equilibrium value n_0 in accord with the formula

$$n = n_0 + \chi \frac{\partial n_0}{\partial \varepsilon},$$

ε is the phonon energy, c the speed of sound, τ the phonon free-path time, v the velocity of the liquid, and $\vec{n}$ a unit vector along the phonon momentum.

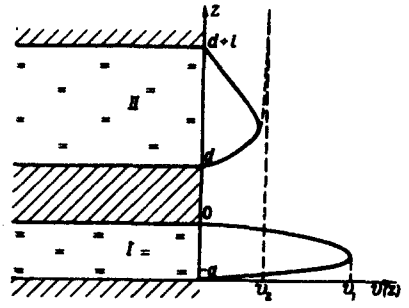

Fig. 1

We assume that the coefficient W of penetration of the phonons through the solid wall is

small. This enables us, when solving Eq. (1) in region (I), to assume that the reflection coefficient is equal to unity and write the boundary conditions for the function χ at z = 0 and -a in the form $\chi(n_z) = \chi(-n_z)$. By substituting the Poiseuille velocity profile $v(z) = v_1[1 - (1 + 2z/a)^2]$ in the right-hand side of (1) and its solution with the given boundary conditions, we can readily determine the value χ_0 of the function χ at z = 0:

$$\chi_0 = -v_1 \frac{8\epsilon\tau}{a^2} n_x n_z \left\{ \frac{a}{2} - cn_z\tau + \frac{a}{\exp(a/cn_z\tau) - 1} \right\}. \qquad (2)$$

We assume for simplicity that the thickness ℓ of layer II is much larger than a. Then the velocity profile in the main part of the volume II is linear, and it suffices to consider the case $\ell = \infty$ for its complete determination (see Fig. 1; the velocity v_2 coincides with the velocity at $z = \infty$ if $\ell = \infty$).

At $\ell = \infty$ the function χ in region II satisfies Eq. (1) without the right-hand side and the condition $\chi(z = d, n_z > 0) = \chi_0 W$, from which we get $\chi(n_z < 0) = 0$ and $\chi(n_z > 0) = \chi_0 W \exp[(d - z)/cn_z\tau]$. Integrating now the equation of motion of the liquid (see [1])

$$\eta \frac{d^2v}{dz^2} = \int \frac{\epsilon^3 d\epsilon\, do}{(2\pi\hbar c)^3} \frac{\partial n_0}{\partial \epsilon} n_x n_z \frac{\partial \chi}{\partial z}$$

with the conditions $v(z = d) = 0$ and $|v(z = \infty)| < \infty$, we obtain

$$v_2 = v(\infty) = \frac{c}{\eta} \int \frac{\epsilon^3 d\epsilon\, do}{(2\pi\hbar c)^3} \frac{\partial n_0}{\partial \epsilon} n_x n_z^2 \tau \chi_0 W, \qquad (3)$$

where η is the viscosity coefficient of the liquid.

The main contribution to the integral in (3) is made by the phonons whose energy is such that the mean free path $c\tau$ is of the order of the liquid-layer thickness a. If the partition thickness d does not greatly exceed a, then the absorption of the phonons in the partition can be neglected in the calculation of W, since sound absorption in a solid is much less than the absorption in a liquid. The condition that W be small means then that the ratio of the acoustic impedances $\rho c/Dc_t$ of the liquid and the solid should be small (ρ and D are the densities of the liquid and solid, respectively, and c_t is the velocity of transverse sound in the solid). The coefficient of phonon penetration through the separation boundary is then small and therefore a phonon entering the solid is reflected many times from both boundaries before an appreciable fraction of the energy leaves the partition. It is therefore clear that the coefficient W from volume I into volume II is equal to (1 - R)/2, where R is the coefficient of sound reflection from the solid-liquid boundary. Using the known expressions for R [3] and for the absorption of sound in a liquid [2], $\tau = \rho\hbar^2 c^2/\gamma\epsilon^2$, where $\gamma = 4\eta/3 + \zeta + \kappa(c_p - c_v)/c_p c_v$, τ is the second viscosity, κ is the thermal-conductivity coefficient, and c_p and c_v are the specific heats

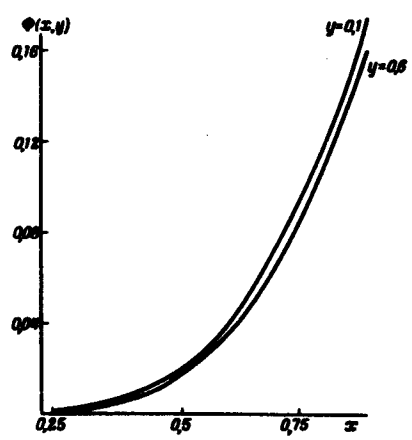

Fig. 2

per unit mass, we obtain from (3)

$$\frac{v_2}{v_1} = \zeta\left(\frac{3}{2}\right)\frac{T}{4\eta}\left(\frac{\rho^3 c}{\pi^5 y^3 a}\right)^{1/2}\frac{\rho c}{Dc_t}\Phi\left(\frac{c}{c_t}, \frac{c_l}{c_t}\right), \tag{4}$$

where

$$\Phi(x, y) = x^4\left\{\int_0^{1/y^2}\frac{(1-y^2t)^{1/2}t(1-x^2t)^{1/4}dt}{y(1-2t)^2+4t(1-t)^{1/2}(1-y^2t)^{1/2}} + \right.$$

$$\left. + \int_{1/y^2}^{1}\frac{4(y^2t-1)t^2(1-t)^{1/2}(1-x^2t)^{1/4}dt}{y^2(1-2t)^4+16t^2(1-t)(y^2t-1)}\right\},$$

c_l is the velocity of the longitudinal sound in the solid, and ζ is the Riemann function. A plot of $\Phi(x, y)$ obtained by numerical calculation is shown in Fig. 2.

In the derivation of the foregoing formula we used for the equilibrium distribution function n_0 its classical value T/ε, since most liquids solidify upon cooling long before quantum effects come into play. We note also that in order for (4) to be valid it is necessary that there be no dispersion of sound in the liquid up to frequencies corresponding to a mean free path equal to a. In other words, the thickness a must be larger than $\rho c^3/\gamma\omega_0^2$, where ω_0 is the frequency above which appreciable dispersion sets in.

For the water-polystyrene (or Plexiglas) pair we obtain from (4) that v_2/v_1 is approximately equal to $10^{-7}/\sqrt{a}$ (a is in cm) at T = 353°K and $10^{-8}/\sqrt{a}$ at T = 293°K; for the mercury-silver pair we obtain approximately $10^{-8}/\sqrt{a}$ at T = 293°K.

The ratio of the total liquid fluxes through regions II and I differs from v_2/v_1, as can be readily seen, by the factor $3l/4a$. On the other hand, if the liquid in region II is at rest, then a pressure gradient is produced in this region, and its ratio to the pressure gradient in region I is equal to $(3a^2/4l^2)(v_2/v_1)$.

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OPTICAL BREAKDOWN OF MOLECULAR GASES

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1. In connection with the progress in the development of high-power lasers (e.g., lasers using molecular gases such as CO_2, CO, etc.) the problem of optical breakdown of molecular gases has become quite vital. The solution of this problem has a direct bearing on the determination of the limiting parameters of lasers for the infrared band.