$$q^*(\text{W/cm}^2) \approx 1.6\cdot 10^{-18}\omega^2 \frac{\Delta}{i} \frac{\langle \hbar\omega_{0m}\rangle_{eV} \sum_m \sigma_{0m}}{\sigma_{tr} \ln[k\cdot 10^{-3}\nu_{eff}\tau / i_{(eV)}]}. \quad (10)$$

We present numerical estimates for molecular nitrogen. In this case, according to [4], $\sum\sigma_{0m} \simeq 3 \times 10^{-16}$ cm^2, $\sigma_{tr} \simeq 1.2 \times 10^{-15}$ cm^2, $\langle\hbar\omega_{0m}\rangle = 0.4$ eV, $i \simeq \Delta \simeq 2$ eV, and at $\tau \simeq 10^{-6}$ sec, $\omega = 2 \times 10^{14}$ sec^{-1} (CO_2 laser), $k = 0.1$, $p \simeq 1$ atm, and $\nu_{eff} \simeq 3 \times 10^{11}$ sec^{-1} we obtain $q^* \simeq 10^{19}$ W/cm^2.

We note that a corresponding estimate, without allowance for the deceleration of the electrons on the vibrational levels in accord to (8), leads under the same conditions to $q \lesssim 10^7$ W/cm^2. Thus, the effect under consideration leads to high power characteristics of molecular-gas lasers.

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TUNNELING OF DISLOCATIONS

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As is well known, the motion of dislocations in the glide plane through the Peierls barriers occurs via production and further expansion of double kink. At sufficiently low temperatures, such a phenomenon has the character of sub-barrier quantum penetration (tunneling). The purpose of the present paper is to calculate the time of tunnel formation of the double kink.

We assume the simplest model wherein the dislocation is represented by a string in a periodic potential field $U_0(y)$ (the Peierls relief). Assume that the string is initially at reat at the bottom of one of the valleys (the zero-point oscillations are assumed small). In the field of a constant external stress F, such a position becomes unstable, and the string will move after a finite average time to the neighboring valley. We shall henceforth consider a potantial relief $U(y) - U_0(y) - Fy$ only for two neighboring valleys. The Hamiltonian of the string is

$$H = \int \frac{\rho}{2}\left(\frac{\partial y}{\partial t}\right)^2 dx + V\{y\}; \quad V\{y\} = \int\left[\frac{\kappa}{2}\left(\frac{\partial y}{\partial x}\right)^2 + U(y)\right] dx. \quad (1)$$

We consider motion only in the glide plane, the coordinate x is directed along the valley, y is the transverse coordinate, ρ is the string density, and κ is its stiffness.

A quantum string is described by a wave function $\Psi\{y\}$, which is a functional of y(x). It is impossible to solve the quantum-mechanical problem in general form. In our case, however, when the height of the barrier U_0 is large compared with the energy of the zero-point oscillation, the quasiclassical approximation is valid. To calculate the transition probability w it is necessary to find the trajectory y(x, t) realizing the given transition, along which the classical action is pure imaginary and has a minimum modulus. The transition probability is $w \sim \exp[2iS/\hbar]$ (we disregard from now on the pre-exponential factors, which are of no importance to us). We obtain a solution of the problem in two limiting cases:

1. Low stresses $F \ll F_p$ (F_p is the Peierls stress, equal to $\max(dU_0/dy)$). In this case the main contribution to the action is made by broad double kinks, which have a practically standard form and differ only in their length ℓ. At sufficiently large ℓ, the potential energy $U(\ell)$ of the double kink can be written in the form

$$V(\ell) = V_0 - Fa\ell \tag{2}$$

(a is the distance between valleys).

Formula (2) admits of an intuitive interpretation. It is natural to regard the dislocation sections located in different valleys as different "phases," the linear energies of which differ by an amount Fa. The energy of the boundaries between the "phases" is equal to $V_0/2$. Such an approach was developed for the case of thermal fluctuations in [1, 2], and for quantum formation of nuclei in a liquid in [3]. Our approach is closest to that of [4].

We can also show that the kinetic energy is connected mainly with the motion of the kink as a whole. The kink "mass" M corresponding to such a motion, as can be seen from the expression (1) for the kinetic energy, is equal to

$$M = \rho \int \left(\frac{\partial y_0}{\partial x}\right)^2 dx \quad . \tag{3}$$

The function $y_0(x)$ describes the equilibrium shape of the kink.

The problem has been finally reduced to one-dimensional form, and the action is calculated in the usual manner

$$S = i \int_0^{V_0/Fa} \sqrt{M(V_0 - Fa\ell)}\, d\ell = \frac{2}{3}\, i\, \frac{V_0^{3/2}}{Fa} \sqrt{M} . \tag{4}$$

We note that formula (2) does not hold near one of the turning points. This region, however, makes no significant contribution to the integral of (4). The transition probability is thus

$$w \sim \exp\left\{- \frac{4}{3} \frac{V_0^{3/2}\sqrt{M}}{\hbar F a}\right\}. \tag{5}$$

To estimate the argument of the exponential, we can use the expression obtained for V_0 in [5], and substitute $\rho a \sim m$ (m is the mass of the atom of the substance) and $\kappa \sim Ga^2$ (G is the shear modulus)

$$\ln w \sim -\frac{F_p}{F}\frac{a\sqrt{mGa^3}}{\hbar}. \tag{6}$$

The multiplier of the ratio F_p/F has the meaning of the ratio of the lattice constant to the amplitude of the zero-point shear oscillations. It is therefore clear that the quantum effects should be sought in substances in which the zero-point oscillation amplitude is not too small. Foremost among these are crystals of inert elements, and also crystals of hydrogen, deuterium, and methane.

There exists a certain temperature region in which the most probable is the complicated process consisting of thermal activation with energy E, followed by tunneling. The probability w of this process is proportional to the product

$$w \sim w_T w_q \sim \exp\left\{-\frac{E}{T} \mp \frac{4}{3}\frac{\sqrt{M}}{Fa\hbar}(V_o - E)^{3/2}\right\}.$$

The most probable value of E at $T > Fa\hbar/4\sqrt{MV_0}$ is

$$E(T) = V_o - \frac{1}{4}\frac{\hbar^2 a^2 F^2}{MT^2}.$$

In this temperature region, the transition probability is

$$w \sim \exp\left\{-\frac{V_o}{T} + \frac{\hbar^2 a^2 F^2}{12MT^3}\right\}. \tag{7}$$

2. Stresses close to the Peierls stresses.

In the case $f = F_p - F \ll F_p$ the potential U(y) near the kink can be represented in the form $U(y) - fy = by^3/3$, where $b = -U_0^m/2$, and the minimum of U(y) is located at the point $y_0 = -\sqrt{f/b}$. The calculation method is similar to that used in [3].

We write down the Hamilton-Jacobi equation for the action, taking as the measurement units $\sqrt{f/b}$ for y, $\kappa^{1/2}f^{-1/4}b^{-1/4}$ for x, and $f(\sqrt{\kappa\rho}/b)$ for S. In terms of these units, the equation takes the standard form

$$\frac{1}{2}\int\left(\frac{\delta S}{\delta y}\right)^2 dx + \int\left[\frac{1}{2}\left(\frac{\partial y}{\partial x}\right)^2 + y - \frac{y^3}{3} + \frac{2}{3}\right]dx = 0. \tag{8}$$

The answer for the transition probability is

$$\ln w \approx -2|S| f \frac{\sqrt{\kappa\rho}}{\hbar b} \tag{9}$$

where |S| is a dimensionless number. An approximate variational estimate yields $|S| \lesssim 20$.

We note that the logarithm of the dislocation velocity is half as large as ln w, a most important fact in the estimate of exponentially small effects.

Attempts to elucidate the role of quantum effects in the overcoming of Peierls barriers by dislocations were made in [6] on the basis of non-rigorous assumptions. These estimates are not confirmed by our calculations, however.

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NEW METHOD OF AMPLIFYING MONOCHROMATIC ULTRASOUND IN A SEMIMETAL

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In [1] there were discussed the conditions under which an intense sound wave of high frequency can strongly drag the carriers of one of the valleys of a semimetal or of a degenerate semiconductor, i.e., produce a particle drift with an average velocity on the order of the sound velocity s. In this communication we assume that a constant electric field E is applied along the wave propagation direction. Within the framework of the model (see below) assumed for the description of this situation, the following result is obtained: the initial wave is amplified (or is damped if the direction of E is reversed), and the entire power jE acquired by the carriers from the field is transferred to the wave (j is the total current of the particles of one valley, generated both by the sound wave and by the field).

Just as in [1], it is assumed that the following conditions are satisfied for the carriers of one valley: 1) A strong magnetic field, sufficient to obtain a one-dimensional spectrum [2], $\hbar\Omega \gtrsim \varepsilon_F$, is applied along the wave propagation direction (Ω is the Larmor frequency and ε_F the Fermi energy). 2) The sound intensity is high enough, $\Delta \gg \hbar\omega$, where Δ is the energy of the interaction of the carriers with the wave and ω is the sound frequency. 3) The sound wavelength λ satisfies the condition $(2\pi\hbar/\lambda) - 2p_F \lesssim p_F(\Delta/\varepsilon_F)$, where p_F is the particle momentum on the Fermi surface. 4) The temperature is low, $kT \lesssim \Delta$. We assume also that $\omega > \nu_n$ (ν_n is the frequency of the collisions with the impurities) and $\Delta \ll \varepsilon_F$.

We use the following model: a one-dimensional gas of charged particles interacts (for concreteness, via a deformation potential with constant Λ) with a classical sound wave and is elastically scattered by the impurities. The corresponding Hamiltonian is

$$\hat{H} = \int dx \left\{ \frac{d}{2}[U_t'^2 + S^2(U_x')^2] + n[\hat{\psi}^+ \Lambda \frac{\partial U}{\partial x}\hat{\psi} + \hat{\psi}^+ \frac{\hat{p}^2}{2m}\hat{\psi} + \hat{\psi}^+ V_{\text{пр}}\hat{\psi} + \hat{\psi}^+ eE\hat{x}\hat{\psi}]\right\}. \quad (1)$$