pump-to-threshold ratios, even without using stable resonators [4]. All this pertains to pulsed operation, when the thermo-optical effects vary in time and ordinary compensation methods are difficult.

By way of an example, Fig. 3 shows photographs of the radiation in the far zone for an ordinary resonator and a waveguide resonator of type 2/2. The garnet crystal operated at a pump-to-threshold ratio of 100 and measured 3 × 6 × 90 mm. Photometry has shown that in the case of the waveguide resonator the divergence decreases greatly and is close to the diffraction limit.

An important property of such resonators is also the homogeneous distribution of the radiation density in the cross section; this increases the strength of the end faces and ensures a narrow directivity pattern.

This permits, for example, to maintain the single-mode regime for a cw garnet laser at large pumping, and by the same token increases appreciably the second-harmonic radiation power.

One more feature of a waveguide resonator, which makes it possible to narrow down the radiation spectrum, i.e., to stabilize the radiation frequency, is connected with the value of the third optical constant W [1], which depends in this case on the resonator parameters and can be compensated for in a sufficiently wide temperature range.

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POSSIBILITY OF ACCELERATING MANY-LEVEL SYSTEMS AND MULTIPOLE-MOMENT SYSTEMS MOVING IN AN INVERTED MEDIUM

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We consider the emission of a quantum $\hbar\omega$ by a system with natural frequencies ω_{ik} moving in a refractive medium with velocity $v = \beta c$. If the system velocity is lower than the phase velocity of the waves in this medium, then the system can emit a quantum only by going from an excited state of energy E_i to a lower state with energy $E_k = E_i - \hbar\omega_{ik}$, corresponding to the normal Doppler effect: $\beta n(\omega) \cos\theta < 1$, where θ is the angle at which the photon is emitted and $n(\omega)$ is the refractive index of the medium. The quantum radiation energy is drawn from the kinetic energy of the system and from the internal energy of the system. Superluminal motion of the system produces the so-called anomalous Doppler effect [1] ($\beta n(\omega) \cos\theta > 1$). The emission of the quantum is accompanied in this case by excitation of the system, and both the excitation energy and the radiation energy are drawn from the kinetic energy of the system. Thus, superluminal motion makes it possible to observe the system in all the states to which it can go over as a result of direct or cascade processes [2, 3].

We wish to call attention to the features of the mechanism whereby a superluminal system interacts with an inverted-population medium. These features cause such a medium to differ greatly from an ordinary medium. We describe the inverted medium by a linear two-level approximation [4], assuming that its dielectric constant is equal to

$$\epsilon(\omega) = 1 \pm \frac{\omega_p^2}{\Omega^2 - \omega^2 - 2i\gamma\omega}, \quad \omega_p^2 = \frac{8\pi d^2 \Omega N}{\hbar}, \tag{1}$$

where Ω is the resonant frequency of the medium, N is the density of the active centers, d is the electric dipole moment, γ is a parameter that takes absorption into account, and the lower and upper signs correspond to the inverted and to the ordinary population, respectively.

We note that wave absorption is characterized by a negative group velocity [5 - 8], and it is seen from (1) that this velocity really becomes negative in the region of the resonant frequency Ω where the condition of the anomalous Doppler effect is satisfied. In the calculation of the energy losses of a radiating charged particle moving in a medium with normal population, retarded potentials are chosen as the solution of the wave equation [9]. The advanced solution is rejected as contradicting the radiation principle and the causality principle. On the other hand, it can be shown that for inverted media it is precisely the inclusion of advanced potentials that leads to a correct expression for the dielectric constant in the form (1) (lower sign). When the concept of operator ε^{-1} ($\varepsilon = \varepsilon\,[-i(\partial/\partial t)]$) is made more precise, in the sense of [9], we find in the case of inverted media that the form of the operator ε^{-1} in the Fourier representation differs from the form of the operator for an ordinary medium in the sign of the δ function

$$\frac{1}{\epsilon(\omega)} \pm i\pi \frac{\omega}{|\omega|} \delta[\epsilon(\omega)]. \tag{2}$$

Taking the foregoing remarks into account, we find that in the linear description of the inverted medium the sign of the energy losses is reversed when the system radiates, i.e., the losses turn into an energy gain. The energy absorbed in a system that is initially in a definite state can be calculated by the usual method [1, 3]. For example, for an atom having a dipole moement $\vec{p}_{ik}$ and oriented along the velocity $\vec{v}$, the energy drawn by the system from the medium per unit time is

$$W = \frac{(1-\beta^2)^3}{c^3} \int \frac{\omega_{ik}^4 |p_{ik}|^2 \sin^3\theta}{(\beta n \cos\theta - 1)^5} d\theta, \tag{3}$$

where $|\vec{p}_{ik}|$ is the matrix element of the dipole transition of the system at rest. The integration with respect to θ is carried out over the angles inside the Cerenkov cone. In (3), we assume that n is constant, which is correct in view of the resonant character of the emission and absorption in a medium with ε given by (1). Since the resonant frequency of the dielectric constant is shifted away from Ω, we can confine ourselves to the following approximate expression for n:

$$n = \sqrt{1 + \frac{1}{2}\frac{\omega_p^2}{\gamma\Omega}}. \tag{4}$$

Integrating, we obtain

$$W = \frac{(1-\beta^2)|p_{ik}|^2 2\gamma\Omega^3}{\beta n c^3}\left[1 - \frac{1}{\beta^2 n^2}\left(1 + \frac{\omega_{ik}}{\Omega}\sqrt{1-\beta^2}\right)^2\right]. \tag{5}$$

We assume that the superluminal velocity of the system (atom) occurs without energy losses (in a channel). The lifetime of the excited levels is of the order of 10^{-3} sec and does not limit the time of flight T of the atom in a medium ∿10 cm thick. For the energy WT obtained by the atom from the inverted medium we get according to (5) a value on the order of 1 keV at the following numerical values of the parameters: $N \sim 10^{18}$ cm^{-3}, $\Omega \sim 3 \times 10^{15}$ sec^{-1}, $\gamma \sim 10^{10}$ sec^{-1}, $d \simeq 1.5 \times 10^{-18}$ cgs esu, and $|p_{ik}| \simeq 10^{-18}$ cgs esu.

We now proceed to consider particles or particle systems having a constant multipole electric or magnetic moment. Such systems moving in an inverted medium with superluminal velocity can draw on the energy stored in the active medium by inverting the Cerenkov radiation force. Formulas for the Cerenkov radiation of such systems were obtained by Frank in [10]. Let us estimate the force acting on a system. This force is determined by the total energy transferred to the system on a unit path length. We integrate formula (3.10) of [10] with respect to the frequency in the frequency region from Ω to $(\Omega + 2\gamma)$, assuming n to be constant and given by (4). As a result we obtain for an electric multipole of order ℓ, oriented parallel ($\theta = 0$) and perpendicular ($\theta = \pi/2$) to the velocity of the moving system, respectively, the following expressions for the accelerating force:

$$\frac{dW^{p\ell\parallel v}}{dz} = \frac{1}{(\ell!)^2}\,\frac{p_\ell^2}{c^2 v^{2\ell}}\left(1 - \frac{1}{\beta^2 n^2}\right)2\gamma\Omega^{2\ell+1}, \tag{6}$$

$$\frac{d|W|^{p\ell\perp v}}{dz} = \frac{(2\ell)!}{(\ell!)^4\, 2^{2\ell}}\,\frac{p_\ell^2 n^{2\ell}}{c^{2(\ell+1)}}\left(1 - \frac{1}{\beta^2 n^2}\right)^{\ell+1} 2\gamma\Omega^{2\ell+1}. \tag{7}$$

The quantity p_ℓ in (6) and (7) (the electric moment of order ℓ) is taken in the laboratory frame.

Formulas for the accelerating force acting on magnetic multipoles are obtained from the corresponding formulas for electric multipoles. This necessitates [10] a replacement of p_ℓ by m_ℓ (magnetic dipole of order ℓ in the laboratory frame) and multiplication of the result by n^2.

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