made concerning the dominant role of the amplitude L^S and if the available data on the photoproduction of X^0 and η mesons on nucleons are used [2, 4]. It should be noted that an independent confirmation of the pseudoscalar character of the η meson is highly desirable.

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LEVEL SPECTRUM OF THREE RESONANTLY INTERACTING PARTICLES

V. Efimov
Leningrad Institute of Nuclear Physics, USSR Academy of Sciences
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A quantum mechanical effect concerning the level spectrum of three particles was reported in [1]. It turns out that the spectrum has a pronounced singularity if the pair forces between particles have a resonant character. In this case, the number of levels can be very large, even infinite in principle. It is necessary for this that the resonance in the pair forces be a bound or virtual state with low energy and zero orbital angular momentum. For three spinless bosons, which were investigated in detail in [2] (see also [3]), the number of levels, with logarithmic accuracy, is

$$N = \frac{|s_0|}{\pi} \ln \frac{|a|}{r_0} \qquad (1)$$

(r_0 is the radius of the forces, and a is the scattering length; $a >> r_0$ for resonant forces; $|s_0| = 1$). In addition, the influence of the Coulomb forces and of the particle spins on the effect was discussed in [2], as well as the possible existence of such levels in the spectrum of three α particles (the C^{12} nucleus) and in the spectrum of three nucleons (H^3).

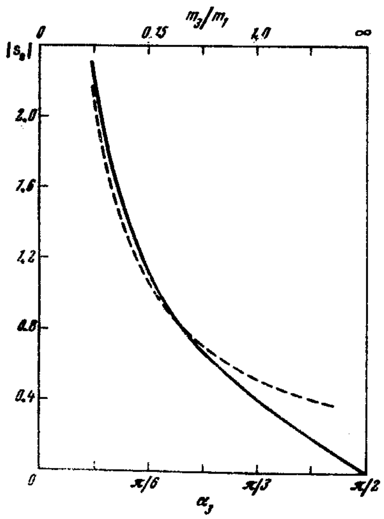

Fig. 1 Plot of $|s_0|$ vs. the particle masses for two resonating pairs; $\alpha_3 = \tan^{-1}(m_3M/m_1m_2)^{1/2}$; M is the total mass of the particles. Upper scale - values of m_3/m_1 in the case when the masses of particles 1 and 2 are equal.

However, the formation of even several levels for three identical particles is quite difficult in practice, since it calls for resonances with large lengths $a_N \sim r_0 e^{\pi N}$. At other mass ratios, the general picture of the effect remains the same as before [4, 5], but the conditions of level formation change both for the better and for the worse. An analysis of this question follows.

Let us consider first the cases more frequently encountered in practice, when only two of the three particle pairs resonate. A plot of $|s_0|$ against the particle masses is shown in Fig. 1. The particle common to the two resonating pairs is numbered 3. The conditions for level formation improve ($|s_0| \to \infty$) when the third particle is much lighter than the other two. At this mass ratio a molecular situation arises, wherein two heavy particles move slowly in the attraction field produced by the fast light particle. The radius of the field is of the order of the smaller of the two scattering lengths, since at larger distances the resonant forces between one pair become ineffective. Calculation shows that the field is given by $U = (0.57)^2/2m_3 r_{12}^2$ at $r_0 \lesssim r_{12} \lesssim a_{min}$, in accordance with the general considerations in [2]. The motion of the heavy particles is classical; the number of levels is

$$N = \frac{1}{\pi} \int_{\sim r_0}^{\sim |a_{min}|} p_{12}\, dr_{12} = \frac{0.57}{\pi} \left(\frac{m_{12}}{m_3} \right)^{1/2} \ln \frac{|a_{min}|}{r_0} = \frac{|s_0|}{\pi} \ln \frac{|a_{min}|}{r_0} \qquad (2)$$

(m_{12} is the reduced mass of the heavy particles). Therefore $|s_0| = 0.57(m_{12}/m_3)^{1/2} = 0.57/\alpha_3$ as $\alpha_3 \to 0$. This "adiabatic" value of $|s_0|$ is shown dashed in Fig. 1. The potential U was calculated earlier by Smirnov and Firsov [6]; in the adiabatic limit, our calculations of the potential agree with their result. For particles with masses of the same order, $|s_0| \sim 1$, the number of levels is given by the right-hand side of (2).

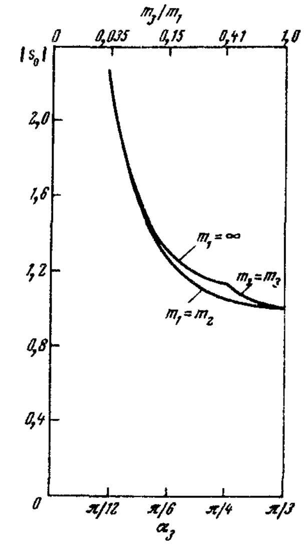

Fig. 2. Dependence of $|s_0|$ on the particle masses for three resonating pairs.

In the limit as $|s_0| \to 0$ the conditions for level formation are made most difficult. We consider it necessary to mention this fact, since an unexpected situation arises here. To illustrate it, we choose particles 1 and 2 to be identical and non-interpenetrating. Then the limit $\alpha_3 \to \pi/2$ corresponds to the case when they move in the field of a center with mass $m_3 \to \infty$. If $m_3 = \infty$, then the spectrum is simple, viz., there is one bound state at $a > 0$. If the mass of the center is finite, however, then its recoil leads, as can be seen, to a spectrum of the type (1), concentrated in a region of very low energies and reciprocal scattering lengths. We obtain the dimension of the region by putting in (1) $N \sim 1$ and $|s_0| \sim m_1/m_3$

$$\Delta(a^{-1}) \sim \frac{1}{r_0} e^{-1/|s_0|} \sim \frac{1}{r_0} e^{-m_3/m_1}. \qquad (3)$$

From this point of view, an infinite number of levels appear and vanish in the spectrum at the instant of occurrence of a two-particle bound state when $m_3 = \infty$.

We proceed to the case when all three pairs resonate. Such a situation is quite feasible, say two neutrons and a nucleus. We assume the third particle to be the lightest and the first to be the heaviest. $|s_0|$ now depends on the two mass ratios. We choose one to be α_3, and the other m_2/m_1. The lower curve of Fig. 2 corresponds to $m_2/m_1 = 1$, and the upper to $m_2/m_1 = 0$. At $\alpha_3 > \pi/4$, the value of $m_2/m_1 = 0$ is not reached, for when m_2/m_1 decreases, m_2 first becomes equal to m_3, and Fig. 2 shows a curve for this case. The values of $|s_0|$ for the intermediate mass ratios m_2/m_1 are located between the curves of Fig. 2. In the adiabatic limit, the curves of Figs. 1 and 2 merge, since the relative momentum in the collisions of heavy particles increases like $m_{12}^{1/2}$, and the condition for their resonance is violated.

Let us calculate the number of levels for masses of the same order (or when one of the particles is heavy). So long as the spatial dimension of the states is smaller than a_{min}, all three pairs resonate. Therefore the number of levels for which $r_{ik} \lesssim a_{min}$ is given as before by the right-hand part of formula (2), but now $|s_0|$ is taken from Fig. 2. At lower energies the character of the spectrum depends on the sign of a_{min}. If $a_{min} < 0$, then the resonant forces of only two pairs are resonant at $r_{ik} \gtrsim a_{min}$. This takes place until the dimension of the states reaches the value of the middle scattering length a_{mid}. The net number of states is therefore

$$N = \frac{|s_0|}{\pi} \ln \frac{|a_{min}|}{r_0} + \frac{|s_0'|}{\pi} \ln \frac{|a_{mid}|}{|a_{min}|} \qquad (4)$$

(with $|s_0'|$ from Fig. 1). On the other hand, if $a_{min} > 0$, then the binding energy of the pair with a_{min} is the limit of the discrete spectrum of the three particles, and only levels with higher binding energy remain in the spectrum. The levels with $r_{ik} \gtrsim a_{min}$ do not satisfy this condition, and therefore the total number of levels is given by the first terms of (4). The same method is used to calculate the number of levels in the remaining case of two heavy particles; the formula is somewhat more complicated and is not given here.

So far we have discussed three-particle 0^+ states. At equal masses, the effect exists only in these states [2]. On approaching the adiabatic limit, the effect arises in states with higher moments [4, 5]. In the adiabatic limit it appears in a state with angular momentum L (and parity $(-1)^L$), when the attraction potential $(0.57)^2/2m_3r_{12}^2$ becomes comparable with the centrifugal energy of the heavy particles $L(L + 1)/2m_{12}r_{12}^2$, i.e., when

$$\frac{m_3}{m_{12}} = \frac{0.32}{L(L+1)} . \qquad (5)$$

It follows therefore that, at a specified mass ratio, the maximum angular momentum for which the effect still exists is $0.57(m_{12}/m_3)^{1/2}$, and it is necessary to substitute in the formula for the number of levels $[|s_0|^2 - L(L + 1)]^{1/2}$ in place of $|s_0|$.

Calculation of the ratio m_3/m_{12} at which the effect appears in the states 1^- and 2^+ yielded 0.15 and 0.05, whereas formula (5) yields 0.16 and 0.05. The agreement is due to the fact that even at L = 1 the mass ratio is small, so that the adiabatic approximation gives correct results.

Let us discuss briefly, on the basis of these results, how the described levels can exist in nature. From the point of view of the mass ratio, the most favorable system is an electron plus two neutral atoms - a negative molecular ion (if the atoms are not neutral, then the Coulomb forces destroy the effect). Here $|s_0|$ ranges from 17 to 270 over the periodic system. In practice, the possibility of level existence is determined both by the real values of the length of electron scattering by each atom, and by the behavior of the terms of the neutral molecule [6, 7].

In nuclei, the possibility of formation of levels is made difficult by the Coulomb forces [2]. A rough estimate of their influence shows that the levels are possible in nuclei with $A \lesssim 20$. An exception is a system of two neutrons plus a nucleus, where there is no limitation on A. For the realized mass ratios we have $|s_0| \lesssim 1$, so that three particles can form in practice one level of the discussed type; its configuration is 0^+. In this nucleus, the state differs from the usual nuclear states in that it is strongly clustered on these three particles and has larger dimensions. It lies energywise near the threshold of three-particle disintegration.

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DIFFRACTION PRODUCTION OF A_1 MESON

Yu.N. Kafiev and V.V. Serebryakov
Mathematics Institute, Siberian Division, USSR Academy of Sciences
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The problem of diffraction reactions and the role played in them by the pomeron (P) has recently attracted special interest. In particular, there is no explanation for the smallness of the diffraction-dissociation (DD) cross sections (∿100 μb, whereas the corresponding elastic cross sections equal 4-6 mb). It is natural to explain these and other properties of DD by means of a conservation law that distinguishes the vertices of the diffraction production from the elastic vertices. This circumstance agrees with the presence of a structure in the differential distribution in dissociative production, for example in the production of N*(1400) and A_1(1080) one observes sharp forward peaks [1] with slopes 12 - 15 GeV^{-2} at small t. These properties of the DD can be explained within the framework of the tensor dominance model (TDM) [2], according to which the vertices of the pomeron and of the f meson are proportional to the corresponding matrix elements of the energy-momentum tensor, and consequently satisfy the conservation law

$$(p_a - p_b)_\alpha < b \,|\, \theta_{\alpha\beta}(0) \,|\, a > \; = 0. \tag{1}$$

In a diffraction transition of a pion into the state A_J ($J^P = 1^+, 2^-, \ldots$), the form factors of this matrix element, which are important for reggeization, turn out to be proportional to $t = (p_a - p_b)^2$. For a spin $J < 2$, this is a kinematic consequence of (1). When $J \geqslant 2$, a factor t appears when account is taken of the ρ - f exchange degeneracy [3], and an analogous conclusion with allowance for the ω - f exchange degeneracy can be made also in the case of baryon vertices. We assume that the P- and f-Regge contributions are proportional [4]. Then the presence of t in the Regge residues leads to a suppression of the DD cross section by a factor $(bm_a^2)^{-2} \sim 10^{-2}$ in comparison with the elastic cross sections (the amplitudes are parametrized as usual: $T = Ae^{bt/2}$, $b \sim 10$ GeV^{-2}). At very high energies the model predicts a decrease of the DD cross sections like $\ln^{-3}s$ and the appearance of dips at t = 0. However, at the presently accessible medium energies (p_L = 10 - 30 GeV/c) an important role is played by the contribution made to the imaginary part of the production amplitude by the two-reggeon branch cuts (see Fig. 1).