Effect of resonant-particle distribution on the dispersion of Bernstein–Green–Kruskal waves

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The influence of the distribution of the resonant particles on the dispersion of the Bernstein–Green–Kruskal waves is investigated for the first time. A theoretical explanation of the observed phenomena is given.

Bernstein-Green-Kruskal (BGK) waves is the name usually given to waves containing a noticeable number of captured particles. The influence of the captured particles on the dispersion and waveform of the waves was the subject of a number of theoretical studies.[1-6] Long ago, Bohm and Gross[1] have determined the correction that must be introduced in the dispersion law to account for the captured particles, but for a special distribution of the latter. Bernstein, Green, and Kruskal[2] have shown that the distribution function of the captured particles can be chosen such as to obtain any prescribed form $\phi(\xi)=\phi(x-v_{ph}t)$ at any arbitrary wave-propagation velocity v_{ph}. The later papers[3-6] dealt also with the influence of the captured-particle distribution on the wave dispersion, but this question has not been studied experimentally to date.

The main difficulty, namely the excitation of BGK waves with a controllable resonant-particle distribution function, was overcome by us through the use of a multi-beam system. Particles are called resonant if their total energy in the rest system of the wave, $w=m(v-v_{ph})^2/2+e\phi$, does not exceed $1.1\,e\phi_s$ (ϕ_s is the wave amplitude). It is precisely the distribution of these particles which determines the dispersion and amplitude of the stationary wave.

We consider the simplest variant of a multibeam system, when a control beam of lower density ($n_2<n_1$), whose velocity is varied in a wide range, is injected into the plasma in addition to the main beam. The initial distribution function of the two beams, having average velocities v_1 and v_2 with $v_2<v_{ph}^0$ (v_{ph}^0 is the initial phase velocity of the wave) and with $(v_1-v_{ph}^0)\sim(v_{ph}^0-v_2)$, can be approximated in the interval (v_1, v_2) (see Fig. 1) by the second-order curves

$$f_0=\frac{1}{2}(v-v_{ph})^2[f_1''\theta(v-v_{ph}^0)+f_2''\theta(v_{ph}^0-v)], \quad (1)$$

where f_1'' and f_2'' are determined from the normalization condition

$$\frac{1}{2}f_{1,2}''\int_0^{v_{1,2}+\Delta_{1,2}}(v-v_{ph}^0)^2dv = n_{1,2}. \quad (2)$$

Here Δ is the half-width of the beam-velocity spread.

From (1) and (2) we can approximately determine (in analogy with the procedure used in[4,5]) the distribution function of the resonant particles in the steady-state wave. We can then obtain from a dispersion equation that follows from the Vlasov and Poisson equations, by calculations similar to those in[4,5], the following expression for the correction to the phase velocity $\Delta v_{ph}(\phi_s)$

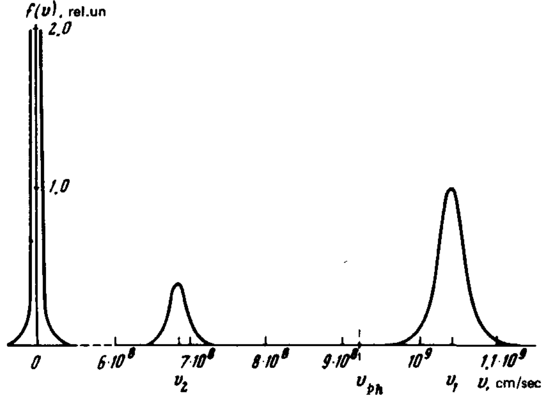

FIG. 1. Initial beam-velocity distribution functions.

$$= v_{ph}(\phi_s) - v_{ph}^0,$$

$$\Delta v_{ph}(\phi_s) = -G\frac{m}{e}\frac{\omega_{pe}^2(v_{ph}-v_{gr})}{k^2}\frac{n^r}{n_0\phi_s}, \quad (3)$$

where $n^r = n_1^r + n_2^r$ is the density of the resonant particles, v_{gr} is the group velocity, n_0 is the plasma density, and G is a factor on the order of unity, which depends on the details of the evolution of the distribution function and does not lend itself to an analytic evaluation.

The quantities ϕ_s, n_1^r, n_2^r, and $v_{ph}(\phi_s)$ are connected also by a relation that follows from the energy conservation law:

$$\phi_s^2 = \frac{8\pi m}{k^2}\{n_1^r[v_1^2 - v_{ph}^2(\phi_s)] + n_2^r[v_2^2 - v_{ph}^2(\phi_s)]\} \quad (4)$$

from which we see that the amplitude ϕ_s depends significantly on the initial velocity v_2 and on the density n_2 of the control beam, and can be small, on the order of the capture amplitude of the main beam[1)]:

$$\phi_{st} \approx 2^{-3}\left(\frac{m}{e}\right)v_1^2\left(\frac{n_1}{2n_0}\right)^{2/3}. \quad (5)$$

It follows from (3) that the change of the phase velocity is proportional to the density of the resonant particles

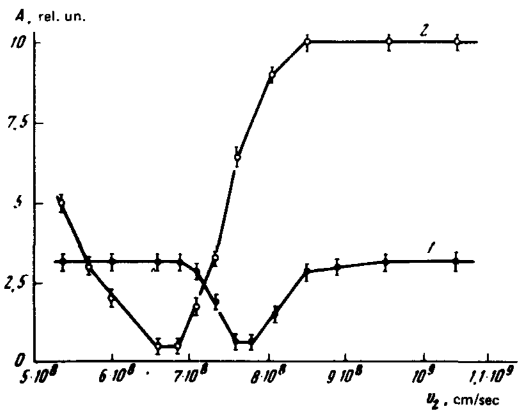

FIG. 2. Wave amplitudes vs control-beam velocity: 1) $n_1 + n_2/n_0 = 0.5\times10^{-3}$, 2) $n_1+n_2/n_0 = 1\times10^{-3}$.

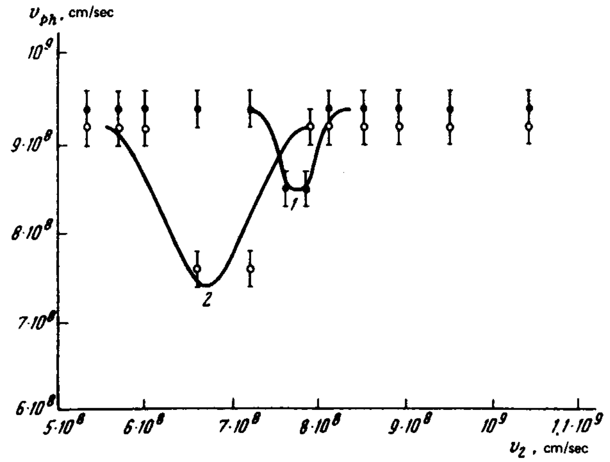

FIG. 3. Experimental values and theoretical plots of the phase velocity vs the control-beam velocity: 1) $n_1+n_2/n_0 = 0.5\times10^{-3}$, 2) $n_1+n_2/n_0 = 1\times10^{-3}$.

and is inversely proportional to the amplitude of the steady-state wave. In particular, in a one-beam system, when $\phi_s^2 \sim 8\pi mk^{-2}n_1v_1^2(n_1/2n_0)^{1/3}$, the correction to the phase velocity of the steady-state wave is negligibly small. This result was obtained also in[4].

The experiments were performed with the setup described in[7]. Two electron beams were injected into a plasma cylinder situated in a strong magnetic field ($\omega_{He} \gg \omega_{pe}$), and a monochromatic signal ($f = 230$ MHz) was applied to the feed electrode near the cathodes and determined the frequency of the wave excited by the beams. To exclude the influence of the low-frequency ion oscillations (which are observed in the frequency range 5—100 kHz), these oscillations were suppressed by applying a small positive potential on the collector. The phase velocity of the wave was measured by comparing the phases of the reference signal and the signal picked up from the plasma by a probe. The ratio of the beam densities was kept constant in the experiment ($n_1/n_2 = 3$), while their summary density could be varied. Figures 2 and 3 show experimental plots of the amplitude and of the phase velocity of the wave against the initial velocity v_2 of the control beam at a distance 34 cm from the start of the plasma column and at $(n_1+n_2)/n_0 = 1\times10^{-3}$ and 0.5×10^{-3}, respectively. It is seen from Fig. 2 that the wave amplitude is strongly limited by the capture of the slow control beam by the wave. The difference between the amplitude levels outside the limitation region ($v_2 \approx 8.5\times10^8$ cm/sec) is attributed to the fact that at the lower summary beam density (curve 1 of Fig. 2) the wave amplitude does not reach saturation within the length of the column. It can be verified that relation (4) describes well the dependence of the amplitudes on $n_{1,2}$ and $v_{1,2}$ (see[7]).

Noting that under the experimental conditions we have $v_1 \approx 10^9$ cm/sec, $k \approx 1$ cm^{-1}, $\omega_{pe} \approx 10^9$ sec^{-1}, and $v_{gr} \approx v_{ph}/2$, we can easily estimate from (3) the value of Δv_{ph} at $\phi_s = \phi_{st}$ (5), i.e., when the amplitude of the steady-state wave is minimal. At $n^r \approx n_1 + n_2 = 10^{-3}n_0$ we get from (3) and (5) $\Delta v_{ph}(\phi_{st}) \approx -4G\times10^8$ cm/sec. The experimentally measured value (Fig. 3), $\Delta v_{ph} = 2\times10^{-8}$

cm/sec, agrees in this case with the obtained estimate if we put $G=0.5$. For the case when the total beam density is half as large, as follows from (3), Δv_{ph} should be approximately half as large. This is confirmed by the measurements (Fig. 3).

Figure 3 (curves 1 and 2) shows the values of $v_{ph} = v_{ph}(v_2)$ obtained by substituting in (3) the experimentally measured function $\phi_s = \phi_s(v_2)$ (Fig. 2). It was assumed that $n^r \sim n_1 + n_2$, and that G is a constant chosen such that curve 1 passes through one of the experimentally measured points. We see that there is a good agreement between the measurements and the obtained theoretical estimates.

[1)]A detailed investigation of the dependence of the amplitude of steady-state BGK waves on the initial beam densities and velocities in a multibeam system is the subject of[7].

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